What to Put on the Table*

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Abstract

This paper investigates the circumstances under which negotiating simultaneously over multiple issues or assets helps reduce inefficiencies due to the presence of asymmetric information. Consider the case where one agent controls all assets. One would expect that strong substitutabilities among assets, would help trade, as the seller would value the additional units less. We show that this intuition is true only if assets are heterogeneous. If assets are homogeneous, efficiency is never possible, irrespective of the degree of substitutability or complementarity among them. When ownership is dispersed, in the sense that different assets are owned by different agents, efficiency is actually more common when assets are homogeneous. When assets are heterogeneous, efficiency can be possible only when assets are complementary. We also study cases where co-ownership is possible (partnerships), allowing for asymmetric distributions, general valuation functions and multiple assets. We show that efficient dissolution is possible if all agents' payoffs at their valuations where gains of trade are minimal are equal. For this to hold, the agent that most likely has the highest valuation for a given asset should initially own a bigger share of that asset. We discuss implications of these findings for the design of negotiation agendas, partnerships and joint ventures. JEL classification codes: C72, D82, L14. Keywords: *efficient mechanism design, multiple units, complements, substitutes, ownership structure, partnerships.*

Many important economic and political decisions are determined through negotiations: They determine the terms of firm acquisitions,[1] of mergers, and of labor contracts and play a key role in international treaties, constitutional reforms, and dispute resolutions. There are usually multiple issues at stake and money often changes hands, as in the cases of M&As and of labor contracts. An important economic insight is that the presence of asymmetric information seriously hinders the ability of negotiating parties

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[1] Recent empirical work by Boone and Mulherin (2007) suggests that about half of company sales are performed via negotiations.

to achieve mutually beneficial agreements. The seminal paper by Myerson and Satterwhaite (1983) shows in a bilateral trading environment with double-sided asymmetric information that no feasible ex-post efficient negotiation procedure exists when gains from trade are uncertain. For this reason, asymmetric information is viewed as a serious form of transaction costs in Coase's tradition.

In many situations, negotiating parties have the option to put more than one issue on the table: In multilateral trade negotiations, a large number of issues are discussed simultaneously. In complex mergers, the ownership of many assets is on the table at the same time. Another example is labor markets for professional sports players. There, the ownership of assets (players' rights) is determined via negotiations, which usually involve multiple players and cash. Each team's valuation for a particular player is private information; players are heterogeneous across multiple characteristics and, most importantly, present strong complementarities and substitutabilities with each other. All these facts make them sometimes expendable for one team, but critical to the success of another. Then, what forces determine whether they can be efficiently allocated given the presence of asymmetric information? Perhaps surprisingly, the economics literature so far has very little to say about such multi-issue negotiations, despite the fact that single-issue negotiations are more the exception than the rule.

The goal of this paper is to study multi-issue negotiations under the presence of asymmetric information, complementarities and substitutabilities among assets, and to ask under which circumstances efficiency is possible. In our formulation, an agent's payoff from a given settlement is a function of his private information (which can be multidimensional). In order to investigate whether there is any conceivable negotiation procedure that leads to efficiency, we use tools of the mechanism design literature. Formally, we ask: In which negotiation scenarios can we expect to find incentive-feasible mechanisms that satisfy voluntary participation without outside transfers? The answer to this question provides insight into how to design the agenda of negotiations (what to put on the table) if the goal is efficiency.

To understand how putting more issues on the table can help, let us first consider the case of two teams negotiating over a single player, whose rights are owned by one of them. From Myerson and Satterthwaite (1983), we know that ex-post efficient trade is impossible. But what if the seller owns, for instance, two forwards and has to bench one of them (so the marginal utility for the second player is lower than for the first)? These log-jams are quite common. Consider, for example, Barcelona's 2007 soccer team and the situation involving Henry and Eto'O who played at the similar positions. Can inefficiencies be reduced if both players are negotiated simultaneously? To what extent, and under which circumstances, do substitutabilities help reduce inefficiencies?

Our first result establishes that, when one agent controls all assets and they are homogeneous, efficiency is never possible, irrespective of whether assets are complements or substitutes. If, however, assets are heterogeneous, as in the case of the soccer players mentioned earlier, then efficiency can be feasible when they are substitutes. Now, consider the case where ownership is dispersed, in the sense that different assets (or issues) are owned (or controlled) by different agents. Then, efficiency for homogeneous assets can be feasible both when assets are complements or when they are substitutes. However, efficiency for heterogeneous assets, can be feasible only if assets are complements. The key difference between the cases of homogeneous and of heterogeneous assets is the dimensionality of private information.

These results suggest that the multi-dimensionality of private information reduces the information rents when ownership is concentrated, while it increases them when ownership is dispersed. What can account for this difference? To answer this question, we identify two key forces that determine whether or not efficiency is feasible: the status quo allocation and the characteristics of the assets. The interplay of these two determines the surplus created from trade, as well as the agents' outside options, both of which are crucial for efficiency to be feasible. The level of outside options matters most at the agents' "critical types." These are the types where gains from trade are minimized, and agents are the most reluctant to participate. Let us see now how the effect of the status quo differs with the dimensionality of private information. Suppose that ownership is concentrated, so we can speak of the agent who owns everything as the seller, and that the assets are homogeneous. Then, the type of the seller where gains from trade are minimal is his highest valuation. This implies a very high outside option, and we get an impossibility, regardless of the complementarity or substitutability of the assets. This echoes the Myerson and Satterthwaite (1983) theorem. On the other hand, if private information is multi-dimensional, and assets are substitutes, along the dimension of the asset with the lower marginal utility, the critical type can be interior. This relaxes the rents and we get efficiency. Now, when ownership is dispersed, both agents can be sellers or buyers. With homogeneous assets, critical valuations can be interior, which relaxes the transfers needed to achieve efficiency. However, if assets are heterogeneous, each owner of an asset is a seller just for that asset and a buyer for the other one, making the corresponding critical types the highest and the lowest valuation.

In the paper, we also study how the subsidy that a broker should provide in order to make efficiency possible under voluntary participation varies with the complementarity or substitutability of the assets. We see that the effect is complex and often non-monotonic. For example, in the case of concentrated ownership, it would be natural to expect that, as the degree of complementarity between assets increases, the deficit incurred also increases since the seller's bundle becomes more valuable to him. While this intuition is true for complements, it fails to hold for substitutes. Sometimes, as issues become less substitutes the deficit decreases. Why is this so? Because less substitutability also means a "bigger pie,"and it can increase the amount a buyer is willing to pay in an incentive-compatible mechanism.

Going back to our sports team example, our findings suggest that in the presence of a log-jam, teams are more likely to find an efficient parting with some of the players. Putting multiple players on the table helps to achieve efficiency when they are substitutes since players are heterogeneous. Moreover, negotiations between teams that own complementary players also help efficiency.

In the previous discussion, we have considered cases where ownership is exclusive. What happens when the ownership of assets is non-exclusive? Co-ownership is common not only in partnerships and in families:

As sport aficionados know, partial ownership of soccer players is quite common. European soccer teams often form partnerships consisting of an upper-division and a lower-division team in order to acquire very young foreign soccer players. By doing so, the top team avoids higher prices in the future, and while the player is too young and inexperienced for the upper division team, he is still useful to the lower level team.

Unfortunately, this strategy has often backfired. Once the player develops and it is time to dissolve the partnership and cash in, the teams often fail to reach agreement. The breakdown of negotiations sometimes even forces a player to miss months of play, which leads to an enormous loss of value. This was exactly the case of Luis Jiménez, a Chilean forward owned in equal shares by Fiorentina (first division) and Ternana (third division). Negotiations broke down for months, and only after a third team (Lazio) stepped in and offered cash in return for borrowing the player, was Ternana able to buy the remaining half shares, allowing the player to continue his career.

The work by Cramton, Gibbons and Klemperer (1987) shows that if a group of ex-ante identical agents jointly own an asset in equal (or close to equal) shares, then it is possible to give complete control to the partner with the highest ex-post valuation. They conclude that similar property rights are a key factor in determining whether or not efficiency is achievable. This role of property rights has also been stressed in the context of public good settings by Neeman (1999) and Schmitz (2002), among others.

What was the problem, then, in the negotiations over Luis Jiménez? If property rights were equal, shouldn't the intuition from Cramton, Gibbons and Klemperer (1987) hold and ex-post efficiency be possible? Was there any better way to structure the partnership? Our subsequent analysis highlights the importance of asymmetries of negotiating parties. We study a partnership environment with asymmetric distributions, general valuation functions and multiple assets. In order to focus on the ownership issue, we abstract from substitutabilities and complementarities. We show that efficient dissolution is possible if all agents' payoffs at their critical type[2] are equal- that is, $\pi_i(v_i^*) = \pi_j(v_j^*)$. For the standard case of linear valuations and symmetric distributions (as in Cramton, Gibbons and Klemperer (1987)), this condition reduces to equal property rights. However, for asymmetric environments, we show that the property rights that guarantee $\pi_i(v_i^*) = \pi_j(v_j^*)$ can be extremely unequal. Moreover, we show that agents who most likely have the highest valuation, should initially own a bigger share of the asset.

Technically, the key force behind these results is the following: By increasing an agent's property rights, the transfer that he must receive in order to guarantee his participation must also increase. Moreover, as we prove in the paper, the marginal increase in the transfer that must be given to an agent if his property rights are marginally increased is exactly equal to his payoff at the critical type $\pi_i(v_i^*)$. Then, when the agents' payoffs at their critical types are different, there is an obvious way to reduce the transfers: by reducing the property rights of an agent with a high valuation at the critical type, and redistributing it to an agent with a lower one. Therefore, if at an ownership structure it holds that $\pi_i(v_i^*) = \pi_j(v_j^*)$ for any pair

[2] This is the type where gains from trade, as measured by the difference between participation and non-participation payoffs, are minimized.

of agents, then this ownership structure is transfer-minimizing. We then show that the transfer-minimizing partnership is efficiently dissolvable.

In Luis Jiménez's case, the problem might just have been the *equality* in ownership shares. Fiorentina was much more likely to value the player more (playing in Series A there is more at a stake), but owned only 50 percent of the shares. The same could generally be said about other joint-ventures. The ones likely to benefit the most ex-post (for example, the ones with more marketing muscle) should own bigger shares of the project. If this is not true, it may be impossible to find a mechanism that transfers control efficiently to one of the participants.

These findings can also shed light on the problem of efficient allocation of new technologies among various firms. This is of great economic relevance because, for technologies, the opportunity cost of misallocation is very high. In a patent race, initial property rights can be seen as the probability that each agent will win the race if no partnership is developed. Our previous result indicates that to have efficient trade, the firms that are better at inventing (that is have the bigger "initial share" r_i) should also have a higher capacity for developing applications *after* the technology is discovered (a better distribution of valuations). This is often not true, as can be seen in the case of the technology used in BlackBerry mobile devices. Research in Motion (RIM), the developer of BlackBerry, did not own the rights to the technology and fought a costly litigation for more than three years with NTP. NTP owned the rights to the technology, but it is primarily a patent-owning company, with no ability to directly develop products and profit from the patents.[3] Our results suggest that in order for efficiency to prevail, firms that are more likely to value the invention more, must also be the ones more likely to develop it. This could provide a rationale for the integration of research departments into big firms. Integration may help avoid lost profits due to transaction costs associated with incomplete information.

Related Literature This paper relates to the enormous literature on efficient mechanism design, which includes the seminal papers by Vickrey (1961), Clarke (1971) and Groves (1973). A significant fraction of this literature is concerned with the design of efficient *trading* mechanisms. The seminal contribution here is Myerson and Satterthwaite (1983). Important extensions, with methodological developments from which we borrow extensively, are in the papers by Makowski and Mezzetti (1993,1994), Williams (1999), Krishna and Perry (2000) and Schweizer (2006). None of these papers investigates the role of complementarities or substitutabilities vis-a-vis the status quo for efficiency. Recently, Segal and Whinston (2009) show that when the status quo is equal to the expectation of the efficient allocation efficiency is possible. We ask a different question: Given the characteristics of assets and the status quo, when is efficiency possible? We often identify possibility in cases where the status quo is different from the expectation of the efficient allocation.

[3] There is extensive press coverage of this lawsuit. For a sample, see "Detractors of BlackBerry See Trouble Past Patents," *The New York Times*, March 6, 2006.

Our results also differ in spirit from those of Fang and Norman (2006) and Jackson and Sonnenschein (2007). Those papers investigate the extend to which the inefficiencies identified by the aforementioned work on trading mechanisms can be alleviated by linking together a large number of independent decisions. Those papers show how one can exploit the law of large numbers, and the ex-ante Pareto efficiency of the desired social choice function to achieve approximate efficiency. The idea of linking independent decisions, which is the main force behind those two papers (and some earlier works mentioned therein), is different from the forces in this paper. Here, we look at a small number of issues and investigate the joint role of their characteristics (whether they are complements or substitutes) and the initial ownership structure for efficiency.

This paper is also related to the literature on partnerships, which stems from Cramton, Gibbons and Klemperer (1987). The analysis of interdependent valuations with one-sided incomplete information is in Jehiel and Pauzner (2004). More recent contributions are the papers by Ornelas and Turner (2007) and Ferreira, Ornelas and Turner (2007), which separate the issue of ownership from control, and the paper by Brusco, Lopomo, Robinson and Viswanathan (2007), who examine an interdependent value environment and allow for non-cash payments.

We proceed to describe our model of negotiations.

1. A model of negotiations

There are I risk-neutral agents negotiating over k issues (or assets). An outcome $z \in Z$, where Z is finite, specifies how the issues are resolved. Agent i's payoff from outcome z is $\pi_i^z(v_i)$, where $v_i = (v_i^1, ..., v_i^k)$. Hence, types are multidimensional and values are private. For all $i \in I$, π_i^z is *decreasing*, *convex* and *differentiable* for all z. We impose no restrictions on how π_i depends on z. This formulation allows for many assets, which may be complements or substitutes. The vector v_i is distributed on $V_i = \times_{k \in K}[\underline{v}_i^k, \overline{v}_i^k]$ according to F_i, with $0 \leq \underline{v}_i^k \leq \overline{v}_i^k < \infty$ for all $k \in K$, and is independent from v_j. We use $F(v) = \Pi_{i \in I} F_i(v_i)$, where $v \in V = \times_{i \in I} V_i$, and $F_{-i}(v_{-i}) = \Pi_{j \neq i} F_j(v_j)$ where $v_{-i} \in V_{-i} = \times_{j \neq i} V_j$. We assume throughout that the distribution F_i has a continuous density function f_i that is strictly positive in its support. It is easy to see that this model contains, as special cases, the environments in Myerson and Satterwhaitte (1983) and Cramton, Gibbons and Klemperer (1987).

Basic Definitions By the revelation principle, we know that any outcome that can be achieved by a bargaining procedure, arises at a truth-telling equilibrium of a direct revelation game. Therefore, we can, without loss of generality ,restrict attention to incentive-compatible direct revelation mechanisms. A *direct revelation mechanism* (*DRM*), $M = (p, x)$, consists of an *assignment rule* $p : V \longrightarrow \Delta(Z)$ and a *payment rule* $x : V \longrightarrow \mathbb{R}^I$.

The assignment rule specifies the probability of each outcome for a given vector of reports. We denote by $p^z(v)$ the probability that outcome z is implemented when the vector of reports is v. The payment rule x specifies, for each vector of reports v, a vector of expected net transfers, one for each agent. The interim expected utility of an agent of type v_i when he participates and declares $\tilde{v}_i$ is

$$u_i(v_i, \tilde{v}_i; (p, x)) = E_{v_{-i}} \left[\sum_{z \in Z} [p^z(\tilde{v}_i, v_{-i}) \pi_i^z(v_i)] + x_i(\tilde{v}_i, v_{-i}) \right]. \tag{1}$$

At an incentive-compatible mechanism we have that $v_i \in \arg\max_{\tilde{v}_i} u_i(v_i, \tilde{v}_i; (p, x))$, and we let

$$U_i(v_i) \equiv \max_{v_i' \in V_i} u_i(v_i, \tilde{v}_i; (p, x)),$$

or

$$U_i(v_i) = E_{v_{-i}} \sum_{z \in Z} [p^z(v_i, v_{-i}) \pi_i^z(v_i)] + X_i(v_i), \tag{2}$$

where $X_i(v_i) = E_{v_{-i}} \left[x_i(v_i, v_{-i}) \right]$.

If negotiations break down because of agent i's unwillingness to participate, allocation $Q_i \in \Delta(Z)$ prevails. The payoff from non-participation is, then, given by

$$\underline{U}_i(v_i) = \sum_z Q_i^z \pi_i^z(v_i), \tag{3}$$

where Q_i^z denotes the probability assigned to outcome z by Q_i . Notice that non-participation payoffs may depend on i's type. If $Q_i \equiv Q$ for all i, we call Q the *status quo*.

The timing is as follows: At stage 0, the designer chooses mechanism (p, x). At stage 1, agents decide whether or not to participate. If all participate, they report their types and the mechanism determines the outcome of the negotiations and the payments. If agent i decides not to participate, Q_i determines the outcome. If two or more decide not to participate, some arbitrary $\{\tilde{Q}_i\}_{i \in I}$ is implemented.

We now provide a formal definition of what it entails for a direct revelation mechanism to be feasible.

Definition 1 *(Feasible Mechanisms) For given outside options $\{Q_i\}_{i \in I}$, we say that a mechanism (p, x) is feasible iff it satisfies:*

(IC) Incentive Constraints
$U_i(v_i) \geq u_i(v_i, \tilde{v}_i; (p, x))$ for all $v_i, \tilde{v}_i \in V_i$ and $i \in I$
(VP) Voluntary Participation Constraints
$U_i(v_i) \geq \underline{U}_i(v_i)$ for all $v_i \in V_i$, and $i \in I$
(RES) Resource Constraints
$\sum_{z \in Z} p^z(v) = 1$, $p^z(v) \geq 0$ for all $v \in V$
(BB) Budget Balance
$\sum_{i \in I} x_i(v) = 0$ for all $v \in V$

Summarizing, feasibility requires that p and x are such that (i) agents prefer to tell the truth about their valuation parameter; (ii) agents choose voluntarily to participate in the mechanism; (iii) p is a probability distribution over Z; and (iv) the mechanism does not generate any surplus or deficit.

Our objective is to investigate what forces that enable the existence of feasible mechanisms that are ex-post efficient.

Definition 2 *A mechanism (p, x) is ex-post efficient iff for all $v \in V$, $p^z(v) > 0$ implies that $z(v) \in \arg\max_{z \in Z} \sum_{i=1}^{I} \pi_i^z(v)$.*

Simply put, an ex-post efficient assignment rule assigns positive probability only to outcomes that maximize the sum of agents' utilities. The *total social surplus* at an ex-post efficient assignment rule is given by:

$$W(v) = \max_{z \in Z} \sum_{i \in I} \pi_i^z(v_i).$$

We now investigate when feasible ex-post efficient mechanisms exist. Very similar conditions have been derived in different setups by McAfee (1991), Makowski and Mezzetti (1994), Williams (1999), Krishna and Perry (2000), and, more recently, by Schweizer (2006). The derivation here is included to facilitate the understanding behind the possibility and impossibility results that we will be establishing later.

From the revenue equivalence theorem,[4] we know that all incentive-compatible mechanisms that implement the same allocation rules generate the same expected payoff for each agent up to a constant. This is. of course, also true for efficient allocation rules. Therefore, the interim information rent of an agent is identical for all incentive-compatible and efficient mechanisms up to a constant. A simple way to calculate the rent is to use a particular class of mechanisms that satisfies these properties, such as the Vickrey-Clarke-Groves class (VCG). In other words, when one needs to investigate properties (such as interim voluntary participation, or ex-post budget balance) of incentive-compatible ex-post efficient mechanisms, the VCG class is a canonical class in that it describes all possible interim payoffs up to a constant. Making the constants large enough is an easy way to satisfy interim voluntary participation (VP), but may break the budget (violate BB). On the other hand, choosing the constants appropriately, one can guarantee BB, but then VP may fail. If both VP and BB are desirable, then it helps to know what are the smallest constants to add to the agents' allocation-dependent part of payoff to guarantee that VP is satisfied. If, at these constants, a surplus is possible, that is, if $\sum_{i \in I} x_i(v) \leq 0$, and assuming free disposal, then budget-balance is possible. In what follows, we formalize these ideas and show how to find the transfer-minimizing VCG, subject to voluntary participation.

[4]See Krishna and Perry (2000) for a general version allowing for multi-dimensional types.

The Transfer-Minimizing VCG As is well known, at a VCG mechanism, an agent's interim payoffs are equal to the expected gains from trade plus a constant; that is,

$$U_i(v_i) = E_{v_{-i}}[W(v)] + K_i. \tag{4}$$

From (4) and (2), it follows the well known fact that, at a VCG mechanism, the payment rule is given by

$$X_i(v_i) = E_{v_{-i}} \sum_{\substack{j \in I \\ j \neq i}} \left[\sum_{z \in Z} [p^z(v) \pi_i^z(v_i)] \right] + K_i, \tag{5}$$

where p^z is an ex-post efficient assignment.

Voluntary participation requires that $U_i(v_i) \geq \underline{U}_i(v_i)$, which, with the help of (4), can be written as

$$K_i \geq \underline{U}_i(v_i) - E_{v_{-i}}[W(v)] \text{ for all } v_i.$$

The type(s) of agent i least eager to participate, is the one where the difference in i's payoffs at the status quo and the gains from trade are the largest that is,

$$v_i^* \in \arg\max_{v_i} \{\underline{U}_i(v_i) - E_{v_{-i}}[W(v)]\}. \tag{6}$$

We choose any of these types arbitrarily and call it the *critical type* of agent i, v_i^*.[5] If K_i is large enough to attract type v_i^*, it will also do so for all other types. Therefore, the *lowest* subsidy that ensures voluntary participation for all types of i, is

$$K_i^* = \underbrace{[\underline{U}_i(v_i^*) - E_{v_{-i}}[W(v_i^*, v_{-i})]]}_{\text{a constant from type } v_i\text{'s perspective}}, \tag{7}$$

with v_i^* given by (6). In fact, the VCG where the payment rule is given by (4) with $K_i = K_i^*$ is the VCG that minimizes the sum of transfers among all VCG (and, hence, among all efficient) mechanisms that satisfy VP because it makes the most reluctant type just indifferent between participating and not.

To see this, note that by substituting (5) with K_i^* from (7), into (2), we get that

$$\begin{aligned} U_i(v_i) &= E_{v_{-i}} \sum_{z \in Z} [p^z(v) \pi_i^z(v_i)] + X_i(v_i) \\ &= E_{v_{-i}} \sum_{z \in Z} [p^z(v) \pi_i^z(v_i)] + \sum_{\substack{j \in I \\ j \neq i}} \left[\sum_{z \in Z} [p^z(v) \pi_i^z(v_i)] \right] + \underline{U}_i(v_i^*) - E_{v_{-i}}[W(v_i^*, v_{-i})] \\ &= \underbrace{E_{v_{-i}}[W(v)] - E_{v_{-i}}[W(v_i^*, v_{-i})]}_{\text{information rents}} + \underset{\text{outside option}}{\underline{U}_i(v_i^*)} \end{aligned} \tag{8}$$

[5] For our purposes, any element of the maximizers will do, because all we are interested in, is the maximal difference $\underline{U}_i(v_i) - W(v)$, which is, by definition, the same for all candidate critical types and for all ex-post efficient assignments when there is more than one.

which together with (6) establishes that at the lowest subsidy K_i^*, the critical type is just indifferent between participating and not; then, it is immediate that every other type participates.

By combining (5) with (7), and by adding over all agents, we find that the lowest possible transfers needed to guarantee voluntary participation are

$$\begin{aligned}\sum_{i\in I} E_{v_{-i}}[x_i(v)] &= \sum_{i\in I} E_{v_{-i}}\left[\sum_{\substack{j\in I\\ j\neq i}}\sum_{z\in Z}[p^z(v)\pi_i^z(v_i)] - W(v_i^*, v_{-i}) + \underline{U}_i(v_i^*)\right] \\ &= \sum_{i\in I}\underline{U}_i(v_i^*) + \sum_{i\in I} E\left[W(v) - E_{v_{-i}}W(v_i^*, v_{-i})\right] - E_{v_{-i}}W(v). \qquad (9)\end{aligned}$$

The VCG with the lowest possible transfers generates a surplus if the sum of transfers that the designer needs to make to the agents is negative- that is, if $\sum_{i\in I} E_v[x_i(v)] \leq 0$, which from (9), is equivalent to

$$E[S(v)] \equiv E[W(v)] - \sum_{i\in I} E\left[W(v) - W(v_i^*, v_{-i})\right] - \sum_{i\in I}\underline{U}_i(v_i^*) \geq 0. \qquad (10)$$

We refer to $E[S(v)]$ as the expected surplus (or deficit): It is equal to the maximized sum of all agents' utilities minus the compensations that agents need to receive. This compensation is in the form of total information rents $\sum_{i\in I} E\left[W(v) - W(v_i^*, v_{-i})\right]$ and outside options $\sum_{i\in I}\underline{U}_i(v_i^*)$. Using a procedure identical to that in Krishna and Perry (2000),[6] one can show a version of their Theorem 2 that states that there exists an efficient, incentive-compatible and individually rational mechanism that balances the budget ex-post iff the VCG mechanism that minimizes the sum of transfers satisfies (10).

As noted by Schweizer (2006), whenever critical types $\{v_i^*\}_{i\in I}$ are such, (10) holds for all type profiles (and not only in expectation), then, the *possibility* result is strong in the following sense: For any distribution of types F that generates the critical types $\{v_i^*\}_{i\in I}$, there exists a feasible and ex-post efficient mechanism. Also, whenever (10) is negative for all type profiles (and strictly negative for a strictly positive measure of types)- that is, whenever

$$S(v) \equiv W(v) - \sum_{i\in I}\left[W(v) - W(v_i^*, v_{-i})\right] - \sum_{z} Q_i^z\pi_i^z(v_i^*) \qquad (11)$$

is negative, we have a strong impossibility result.

The distribution of types matters because, together with the assignment rule, they determine the shape of U_i, which, in turn, together with $\underline{U}_i$, determine the critical types, which are a crucial input of (11). In some specific environments, such as, the one in Myerson and Satterwhaite (1983), the critical types are the same for all distributions: The critical type for the seller is his highest possible valuation, and the

[6] One can also use the more general construction from d'Aspremont, Crémer and Gérard-Varet (2004) or Borgers and Norman (2009).

critical type for the buyer is his lowest valuation. However, in general, different distributions F_i could induce different vectors of critical types v_i^*. In order to carry out the analysis in those cases, it seems that, a priori, it is impossible to avoid having to find out the critical types, which requires the computation of the expectation U_i. This can be especially tedious and cumbersome in multi-dimensional settings like the ones that we examine below. There, one needs to first find the ex-post efficient assignment for each region of valuations and to then integrate agents' payoffs over these different multi-dimensional regions.

In what follows, we show how, even in such a-priori seemingly intractable cases, we can employ (11) to answer the questions we posed in the introduction. In order to do so, we look at the simplest possible scenarios that allow us to investigate the interaction between the assets characteristics (that is, whether they are complements or substitutes, homogeneous or heterogeneous) and the initial ownership structure in achieving efficiency. To that end, we will calculate (11) for a number of special cases of the general environment presented in the first paragraph of this section. In some of these environments, agents' private information is one-dimensional; while in others it is multi-dimensional; payoffs sometimes will be linear in types, etc. The benefit of writing down the general model of this section is that it permits us to obtain the general version of (11), which we then adapt to each of the environments that we consider.

We start by analyzing negotiations under exclusive ownership. We analyze the cases of homogeneous (Section 2.) and heterogeneous assets (Section 3.). In each of these classes, we look at the cases where assets are complements or substitutes and at the cases where ownership is concentrated in the sense that all assets are owned by the same agent- or dispersed- in sense that different assets are owned by different agents. Finally, in Section 4. we examine negotiations in cases where ownership is non-exclusive, or as they are more commonly called, partnerships.

2. Negotiations under Exclusive Ownership I: Homogeneous Assets

This section studies negotiations over multiple homogeneous assets in situations where each asset is owned exclusively by one agent.

There are two agents, 1 and 2, and two identical and indivisible assets. There are three possible allocations: Agent 1 gets both assets: allocation $z_1 = (2, 0)$; each agent ends up with one asset: allocation $z_2 = (1, 1)$; or agent 2 gets both assets: allocation $z_3 = (0, 2)$. The payoffs that accrue to agents 1 and 2 at each of these allocations are respectively given by:

$$\begin{array}{ll} \pi_1^{z_1}(v_1) = (1+\alpha)v_1 & \pi_2^{z_1}(v_2) = 0 \\ \pi_1^{z_2}(v_1) = v_1 & \pi_2^{z_2}(v_2) = v_2 \\ \pi_1^{z_3}(v_1) = 0 & \pi_2^{z_3}(v_2) = (1+\alpha)v_2 \end{array},$$

where v_i, $i \in \{1, 2\}$ is distributed according to F_i on $[\underline{v}_i, \bar{v}_i]$.

When $\alpha < 1$, the marginal utility of owning the second asset is lower than the first, and the assets are substitutes. When $\alpha > 1$, the marginal utility of owning the second unit is higher than the first, and the assets are complements: The second unit is more useful at the margin for an agent who already owns one unit. The units are "unrelated" if $\alpha = 1$ since, in this case, the marginal utility of owning a unit of the asset is independent of the number of units owned.

2.1 Concentrated Ownership

Our first result is a strong impossibility result: It shows that if one agent (say, agent 1) owns both assets, and gains from trade are uncertain, in the sense that $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$, then for all $\alpha > \frac{\underline{v}_2}{\bar{v}_1}$ ex-post efficient negotiation procedures do not exist. This could be viewed as surprising because if α is very small, the owner does not really care about the second unit, which implies an extremely small conflict of interest. Note that our condition that gains from trade are uncertain $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$, is a straightforward generalization of the Myerson and Satterwhaite (1983) condition $\underline{v}_2 < \bar{v}_1$. This modification is relevant for the cases where $\alpha < 1$, where the smallest degree of substitutability that makes the analysis non-trivial is $\alpha > \frac{\underline{v}_2}{\bar{v}_1}$.

Proposition 1 *Suppose that ownership is concentrated, in the sense that the status quo is given by allocation* $(2, 0)$.[7] *Suppose, also, that for* $i \in \{1, 2\}$, v_i *is distributed on* $[\underline{v}_i, \bar{v}_i]$ *with full support, satisfying that* $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$. *Then, there is no ex-post efficient, incentive-compatible and individually rational mechanism that balances the budget for any* $\alpha > \frac{\underline{v}_2}{\bar{v}_1}$.

From Proposition 1, we conclude that, irrespective of the degree of substitutability or complementarity of the two units of the asset, it is not possible to design an ex-post efficient negotiation procedure.

Despite the pessimistic news of Proposition 1, we would like to look a bit further and ask how the degree of substitutability or complementarity of the assets affects these unavoidable inefficiencies. As a measure of the inefficiency, we use the expected surplus or deficit, defined in general by (10), which in the current environment can be parameterized by the degree of complementarity α as follows: $\bar{S}(\alpha) = E_v[S(v, \alpha)]$. Its magnitude equals the transfers that a broker should bring into the system in order to make efficiency under budget balance possible: Then, the higher the subsidy needed, the higher the degree of inefficiency.

Proposition 2 *Suppose that* $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$. *If ownership is concentrated, in the sense that the status quo is given by allocation*[8] $(2, 0)$, *then the expected surplus* $\bar{S}(\alpha)$ *is decreasing in* α *for all* $\alpha > 1$, *whereas it can be non-monotonic in* α *for* $\alpha \in [\frac{\underline{v}_2}{\bar{v}_1}, 1]$.

Proof. Consider $S(v, \alpha)$. By integrating over v, we get $\bar{S}(\alpha) = \sum_{\rho \in \mathcal{Q}} \int_{R_\rho(\alpha)} S_\rho(v, \alpha)dv$, where each region $R_\rho(\alpha)$ corresponds to a different realization of the maxima of (31), and $\mathcal{Q}$ stands for the number of regions

[7] The case where the status quo is $(0, 2)$ is identical after one switches the indexes 1 and 2.

[8] The case where the status quo is $(0, 2)$ is identical after one switches the indexes 1 and 2.

where S takes a different expression. By looking at (31), it is easy to see that there are finitely many of these different regions.

Given that there exists K such that $|S_\rho(v,\alpha)| \leq K$, and $|\frac{\partial S_\rho(v,\alpha)}{\partial \alpha}| \leq K$, and that $S(\cdot,\alpha)$ and $R_\rho(\cdot)$ are continuous, we can write, by familiar envelope arguments, that

$$\bar{S}'(\alpha) = \sum_{\rho \in \mathcal{Q}} \int_{R_\rho(\alpha)} \frac{\partial S_\rho(v,\alpha)}{\partial \alpha} dv.$$

This observation tells us that the indirect effect of α on $\bar{S}'_\rho(\alpha)$ through the change of the regions of the maxima $R_\rho(\alpha)$ is zero, and it allows us to focus only on the direct effect of α on $\bar{S}(\alpha)$.

For $\alpha > 1$, we have the following:

- In the region R_ρ where $v_1 \geq v_2$, we have that $S_\rho(v,\alpha) = 0$.
- In the region R_ρ where $v_1 \leq v_2$ and $v_1 \geq \underline{v}_2$, we have that $S_\rho(v,\alpha) = (1+\alpha)(v_1 - v_2)$.
- In the region R_ρ where $v_1 \leq v_2$, $v_1 \leq \underline{v}_2$ and $v_2 \leq \bar{v}_1$, we have that $S_\rho(v,\alpha) = (1+\alpha)(\underline{v}_2 - v_2)$.
- In the region R_ρ where $v_1 \leq v_2$, $v_1 \leq \underline{v}_2$ and $v_2 \geq \bar{v}_1$, we have that $S_\rho(v,\alpha) = (1+\alpha)(\underline{v}_2 - \bar{v}_1)$.

Since $\frac{\partial S_\rho(v,\alpha)}{\partial \alpha} \leq 0$ for all regions, and it is strictly negative in a region of positive measure, the result follows.

Now, to establish the non-monotonicity result for $\alpha < 1$, it is enough to show that there are regions where $\frac{\partial S_\rho(v,\alpha)}{\partial \alpha}$ is bigger or smaller than zero. If the distribution puts enough mass there, the surplus would be increasing or decreasing in α at a given level.

Consider, first, the region where $\frac{v_2}{\alpha} \geq v_1 \geq \alpha v_2$, $\alpha v_1 \geq \underline{v}_2$ and $\alpha \bar{v}_1 \geq v_2$, then $S_\rho(v,\alpha) = -v_2 + \alpha v_1$. In that case, $\frac{\partial S(v,\alpha)}{\partial \alpha} \geq 0$. Then, consider the region where $\frac{v_2}{\alpha} \geq v_1 \geq \alpha v_2$, $\alpha v_1 \geq \underline{v}_2$ and $\alpha \bar{v}_1 \leq v_2$, $S(v,\alpha) = \alpha(v_1 - \bar{v}_1)$. In that case, $\frac{\partial S(v,\alpha)}{\partial \alpha} \leq 0$. These two observations imply the non-monotonicity of the surplus in α whenever $\alpha < 1$. ■

Proposition 2 is easy to understand as follows: When one agent owns all the assets and $\alpha > 1$, at an ex-post efficient assignment, the agent with the highest valuation should end up with both assets. Then, it is as if we have a Myerson-Satterwhaite scenario with an asset that gives a higher marginal utility. When $v_2 > v_1$, agent 2 (who, under the status quo (2,0)), is the buyer) should get both assets. In this case, the sum of transfers is $(1+\alpha)(v_1 - v_2)$, which is negative and increasing in α. When $\alpha > 1$, the higher the α is- that is, the more complementary the goods are- the more money has to be injected into the system. In other words, substitutability (low α) reduces the sum of outside transfers needed for efficiency but is not enough to balance the budget.

When $\alpha < 1$, in contrast to the case of $\alpha > 1$, it is possible that ex-post efficiency dictates that each agent should end up with one asset. If this is the case ($\frac{v_2}{\alpha} > v_1 > \alpha v_2$) there are regions of valuations where

$S(v_1, v_2, \alpha)$ is increasing in α. For example, if $\alpha\bar{v}_1 \geq v_2$ and $\alpha v_1 \geq \underline{v}_2$ the surplus becomes $S(v_1, v_2, \alpha) = -v_1 - v_2 + (1+\alpha)v_1$, which is clearly increasing in α. This region highlights nicely the potentially positive role of α on the expected surplus: It increases the pie, and in this particular case it increases the payment of agent 2, who must pay αv_1 to agent 1 in order to acquire the first unit. Since the payment demanded by agent 1 is the marginal utility of agent 2, v_2, which is independent of α, we get that the decreasing substitutability actually helps efficiency. Note, however, that this is not always the case: α may decrease the surplus because agent 1's payoff at his critical type, for which he must be compensated, is $(1+\alpha)\bar{v}_1$, and it increases with α. Ultimately, the sign of $\bar{S}'(\alpha)$ depends on the relative size of the various regions.

For the case of uniform distribution, the expected surplus turns out to be decreasing in α for all $\alpha > 0:$ The expected surplus- in this case, deficit- as a function of α is given by $\bar{S}(\alpha) = E_v[S(v_1, v_2, \alpha)] = \frac{2}{3}a^2 - \frac{5}{6}a - \frac{1}{6}a^4$, and is depicted Figure 1:

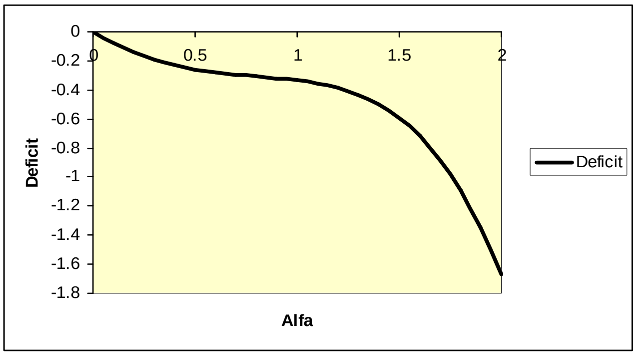


Figure 1: Surplus (Deficit) with Uniform Distribution

However, as our earlier discussion alluded to, in general, expected surplus can be non-monotonic when $\alpha < 1$. An example where this holds is as follows: Suppose that both agents' valuations are distributed according to

$$F(v) = \begin{cases} \frac{\epsilon}{\alpha^2\bar{v}}v & \text{if } v \leq \alpha^2\bar{v} \\ \frac{1-2\epsilon}{\alpha(1-\alpha)\bar{v}}v + \epsilon - \frac{(1-2\epsilon)\alpha}{1-\alpha} & \text{if } v \in [\alpha^2\bar{v}, \alpha\bar{v}] \\ \frac{\epsilon}{(1-\alpha)\bar{v}}v + 1 - \frac{\epsilon}{1-\alpha} & \text{if } v \geq \alpha\bar{v} \end{cases}.$$

For this scenario the derivative of expected surplus with respect to α converges to $\bar{v}(1 - \frac{\alpha(1-\alpha)}{2} - (1-\alpha)) > 0$ as ϵ goes to 0.[9]

Summing up, when one agent owns all the (homogeneous) assets, efficiency cannot be achieved regardless of their degree of complementarity or substitutability. Not surprisingly, a higher degree of complemen-

[9]Details of the calculations for this example are available upon request.

tarity (a higher α) reduces the expected deficit only when $\alpha > 1$; however, more surprisingly, the opposite may be true when $\alpha < 1$. This is because when $\alpha < 1$, it may be ex-post efficient that each agent ends up with one asset, in which case the buyer pays the seller an amount that is increasing in α. On the other hand, when $\alpha \geq 1$, either no units, or both units are traded. Trade takes place when $v_2 > v_1$, in which case the sum of transfers is $v_1 - v_2$ times $(1+\alpha)$, which is decreasing in α.

Do these results hold when each asset is owned by a different agent? This is addressed in the following section.

2.2 Dispersed Ownership

A natural question to ask is what happens if ownership is still exclusive but is dispersed, in the sense that each asset is owned by a different agent. It is easy to see that when $\alpha = 1$, this situation is isomorphic to a scenario where there is one unit and each agent owns half of the property rights. Then, from the work of Cramton, Gibbons and Klemperer (1987), we know that if agents are ex-ante symmetric and each owns equal (or close to equal) shares of an asset, then efficiency is possible. For $\alpha \neq 1$, whether efficiency is possible or not depends on the location of the critical types and on how close α is to zero or to one. For some distributions, as we see below, efficiency is possible even for quite low α, which is interesting because it says that efficiency is possible, even in cases where the marginal value of obtaining the second unit (α) is much lower than the marginal value to the seller.

First, we examine the case where assets are complements, in the sense that $\alpha > 1$. In this case, we see that the forces behind the efficiency result of Cramton, Gibbons and Klemperer (1987) are strengthened.

The Case of Complements ($\alpha > 1$): As we mentioned in the case of concentrated ownership, when $\alpha > 1$, at the ex-post efficient assignment, the agent with the highest valuation should end up with both units. Hence, the situation is very similar to a single-asset scenario, with the difference that the marginal value of the asset is higher. The following Proposition establishes a possibility result when $\alpha \geq 1$ that echoes the possibility result of Cramton, Gibbons and Klemperer (1987).

Proposition 3 *Suppose that ownership is dispersed, in the sense that the status quo is given by allocation* $(1, 1)$*. Suppose, also, that for* $i \in \{1, 2\}$, v_i *is distributed in* $[\underline{v}_i, \bar{v}_i]$ *with full support. Then, if the assets are complements* ($\alpha \geq 1$)*, efficiency is possible for all cases where* $\max\{v_1^*, v_2^*\} \leq \alpha \min\{v_1^*, v_2^*\}$*. In particular, efficiency is possible for all symmetric environments.*

From Proposition 3, we can conclude that efficiency requires that agents' payoffs at their critical types are close. When agents are ex-ante asymmetric, efficiency is possible if critical types are close enough. Moreover this condition is relaxed by the degree of complementarity between the assets. As we will see more transparently when we examine negotiations allowing for co-ownership in Section 4., payoffs at critical types stand for the marginal cost of an agent's participation. The sum of transfers necessary for voluntary participation is, then, minimized when these marginal costs are equalized across agents. As we discuss more extensively in Section 4. this is also the main force behind the possibility result of Cramton, Gibbons and Klemperer (1987).

We now investigate how the degree of complementarity α affects the expected surplus in the case where ownership is dispersed. We show, in contrast to the case of concentrated ownership, that when $\alpha > 1$ and agents are ex-ante symmetric, as the degree of complementarity grows, the surplus generated also grows.

Proposition 4 *Suppose that ownership is dispersed, in the sense that the status quo is given by allocation* $(1, 1)$*. Then, when agents are ex-ante symmetric, the expected surplus* $\bar{S}(\alpha)$ *is increasing in* α *for all* $\alpha > 1$.

Proposition 4 established that when ownership is dispersed, the expected surplus is increasing in α when $\alpha > 1$. This is opposite to what we got for the case of concentrated ownership in Proposition 2. Notice also, the difference in the method of proof with Proposition 2. With concentrated ownership, the critical types are always the same ($v_1^* = \bar{v}_1$ and $v_2^* = \underline{v}_2$). However, when ownership is dispersed critical types vary with α and with the distribution. Moreover the sign of $\frac{\partial S(v,\alpha)}{\partial \alpha}$ changes with v, and the size of the regions of each sign, change with the locations of the critical types. For this reason we can only sign $\frac{\partial S(v,\alpha)}{\partial \alpha}$ only in expectation.

A more substantial difference between the dispersed and concentrated ownership cases, is that with dispersed ownership, in asymmetric environments the surplus can be non-monotonic in α even if $\alpha > 1$. In Appendix B, we present an example where this is true.

In order to understand the reasons behind this non-monotonicity result, it helps to step back a bit. In symmetric environments, we know from the work of Cramton, Gibbons and Klemperer (1987) that $\bar{S}(1) > 0$ and, moreover, we saw that $\bar{S}'(\alpha) > 0$ for $\alpha > 1$. However, in asymmetric environments, $\bar{S}(1)$ can be negative, since critical types can be far apart. Not only that, but the surplus can be decreasing in α for some $\alpha > 1$. The example in Appendix B illustrates exactly that: For two very asymmetric agents, one with low valuation and one with high valuation (on average), the surplus can be decreasing in α, even for $\alpha > 1$. As asymmetry is big, the payment demanded from the high-valuation type (which gets both objects with probability close to 1) is big enough to preclude existence of an efficient mechanism, and this effect can grow with his willingness to pay (indexed by the parameter α).

This concludes our investigation of the existence of efficient negotiations under concentrated ownership, in the case of complements ($\alpha > 1$). We now turn to the case of substitutes ($a < 1$), which is richer, since it is no longer necessarily the case that at an ex-post efficient assignment the agent with the highest valuation ends up with all the units.

The Case of Substitutes: $\alpha < 1$

Proposition 5 *Suppose that ownership is dispersed, in the sense that the status quo is given by allocation* $(1,1)$*. Moreover, suppose that for* $i \in \{1,2\}$*,* v_i *is distributed in* $[\underline{v}_i, \bar{v}_i]$ *with full support, with* $\underline{v}_2 < \alpha \bar{v}_1$*. Then, for any* $\alpha < 1$*, there exist environments (distributions* F_1*,*F_2*), such that efficiency is possible.*

Proof. We establish the possibility of efficiency for a symmetric environment. In such a case, the surplus can be written as

$$\begin{aligned} S(v,\alpha) &= -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} \\ &\quad +\max\{(1+\alpha)v^*, (1+\alpha)v_2, v^*+v_2\} \\ &\quad +\max\{(1+\alpha)v_1, (1+\alpha)v^*, v^*+v_1\} - 2v^*, \end{aligned} \tag{12}$$

where v^* is the critical type of both agents. At that type, participation and non-participation payoffs are tangent, which implies that v^* satisfies $\alpha F(\alpha v^*) + F(v^*/\alpha) = 1$.

Consider a distribution F and a point $\hat{v} \in [\underline{v}, \bar{v}]$ such that $F(\alpha\hat{v}) = \frac{1}{1+\alpha} - \epsilon$ and $F(\hat{v}/\alpha) = \frac{1}{1+\alpha} + \epsilon$. Then, $v^* \approx \hat{v}$ and, with probability close to 1, we have one of the following two cases:

- $v_i \leq \alpha v^*$, in which case $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + 2\alpha v^* \geq 0$
- $v_i \geq \alpha v^*$, in which case because $v^* \approx \hat{v}$ and, with probability close to 1. Moreover, because the values of $v \in [\alpha\hat{v}, \hat{v}/\alpha]$ occur with probability ϵ, this case boils down to $v_i \geq \frac{v^*}{\alpha}$. Then, we immediately have that $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + (1+\alpha)v_1 + (1+\alpha)v_2 - 2v^* \geq 0$. ■

When goods are substitutes ($\alpha < 1$) and ownership is dispersed, equality of critical types across agents is not enough on its own to guarantee possibility. Efficiency is possible, but only for special environments, as in the class we considered in the proof of Proposition 5.

Now we characterize some properties of the expected surplus as a function of the degree of substitutability for the case that $\alpha < 1$.

Proposition 6 *Suppose that ownership is dispersed and that* $\underline{v}_2 < \alpha\bar{v}_1$*. Then, the expected surplus* $\bar{S}(\alpha)$ *satisfies the following properties: (i)* $S(\frac{\underline{v}_2}{\bar{v}_1}) = 0$*. (ii) There exists* α^* *such that* $\bar{S}(\alpha) \leq 0$ *and, therefore,* $\bar{S}(\alpha) < 0$ *for all* $\alpha \in [\frac{\underline{v}_2}{\bar{v}_1}, \alpha^*]$*.*

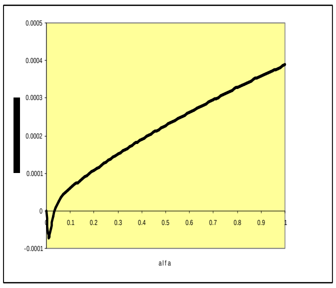
Proof. The first point is direct from the definition of $S(v,\alpha)$. For the second point, fix $\epsilon > 0$. Then, there exists α^* such that, for all $\alpha < \alpha^*$, there is a fraction bigger than $1-\epsilon$ in the region where $\alpha v_2^* \leq v_1 \leq \frac{v_2^*}{\alpha}$ and $\alpha v_1^* \leq v_2 \leq \frac{v_1^*}{\alpha}$. Noting that in that region $S(v,\alpha)$ can take only three values, 0, $-\alpha v_1 + v_2$ and $-\alpha v_2 + v_1$, we get that $\frac{\partial S(v,\alpha)}{\partial \alpha} \leq 0$, the result follows. ■

Contrary to the case of complements ($\alpha > 1$), when $\alpha < 1$, we cannot sign the effect of α on the expected surplus, except for very small values, where a decrease in substitutability definitely hurts the surplus.

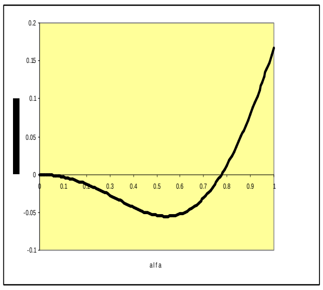
In order to exemplify the ambivalent effect of the degree of substitutability on the surplus, we examine a symmetric scenario where both agents' valuations are distributed on $[0, 1]$ according to

$$F(v_i) = v_i^p, \text{ for } 0 < p < \infty. \tag{13}$$

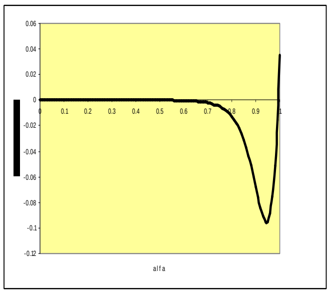
We study how the parameter p (which determines how concentrated the distribution on small values is) affects the threshold level of α needed to achieve efficiency.[10] For this family, there is a cutoff value of α above which efficiency is possible, which is increasing in p. For example, for $p = 0.5$, this cutoff is 0.6, whereas for $p = 0.25$, the cut-off is 0.41. In the following figure, we graph the expected surplus for the uniform case, $p = 0.01$, and for $p = 1$ and $p = 10$:



p=0.01



p=1



p=10

.

The graphs highlight the non-monotonicity of the surplus in α, whenever $\alpha \leq 1$. We can see that as p grows, and high valuations become more probable, there is a higher threshold value for α, above which efficiency is possible. Higher valuation, which makes an agent less likely to be willing to part with his object, makes efficiency more difficult. There a higher α because it increases the willingness to pay for the asset.

[10] When both agents' valuations are distributed according to (13), the expected surplus is

$$\begin{aligned} \bar{S}(\alpha) &= 2\int_{\alpha v^*}^{\alpha} v dF - 2\alpha \int_0^1 v^{p+1}\alpha^p dF - 2\int_0^{\alpha} v^{p+1}\alpha^{-p} dF + \alpha\{1+\alpha^p\} \int_{v^*\alpha^{-1}}^{1} v dF \\ &= 2\frac{p}{p+1}\alpha^{p+1}(1 - v^{*p+1}) - 4\frac{p}{2p+1}\alpha^{p+1} + \alpha(1+\alpha^p)\frac{p}{p+1}(1 - \frac{v^{*p+1}}{\alpha^{p+1}}), \end{aligned}$$

where v^* satisfies $1 = \alpha(\alpha v^*)^p + (\frac{v^*}{\alpha})^p$.

This concludes our analysis of negotiations of multiple homogeneous assets under exclusive ownership. The main lessons are that, when the assets are complements, in the sense that $\alpha > 1$, the situations under which efficiency is possible or not echo the ones of the single-asset world. This is because of the single-dimensionality of valuation parameters and the fact that when $\alpha > 1$, at an ex-post efficient assignment, the agent with the highest valuation should end up with all the objects. The cases where $\alpha < 1$, on the other hand, highlight the complex effect of α on the expected surplus (or deficit). With concentrated ownership, a higher α implies a higher outside payoff to the seller, which has a negative effect on the surplus. On the other hand, a higher α increases the amount that the buyer pays, which has a positive effect on surplus. Now, when ownership is dispersed and $\alpha = 0$, trade never takes place. As α starts to increase, there are realizations of valuations (when $\alpha v_i > v_j$) where trade should occur. In those circumstances, when critical values are high, a deficit occurs. Once α reaches a high enough level, those regions of deficit disappear.

In the following section, we examine negotiations over heterogeneous assets.

3. Negotiations under Exclusive Ownership II: Heterogeneous Assets

Here, we look at two agents who negotiate over two heterogeneous assets A and B. An agent's type consists of two parameters: one for each asset- in other words, types are multidimensional. There are four possible allocations: agent 1 gets both assets, and agent 2 gets none, allocation $z_1 = (AB, 0)$; agent 1 gets asset A, whereas asset B goes to agent 2, allocation $z_2 = (A, B)$; agent 1 gets asset B, and agent 2 gets asset A, allocation $z_3 = (B, A)$; and, finally, agent 1 gets none of the assets, allocation $z_4 = (0, AB)$. The two agents' payoffs in each of these possible allocations are given by:

$$\begin{array}{ll} \pi_1^{z_1}(v_1^A, v_1^B) = v_1^A + \alpha v_1^B & \pi_2^{z_1}(v_2^A, v_2^B) = 0 \\ \pi_1^{z_2}(v_1^A, v_1^B) = v_1^A & \pi_2^{z_2}(v_2^A, v_2^B) = v_2^B \\ \pi_1^{z_3}(v_1^A, v_1^B) = v_1^B & \pi_2^{z_3}(v_2^A, v_2^B) = v_2^A \\ \pi_1^{z_4}(v_1^A, v_1^B) = 0 & \pi_2^{z_4}(v_2^A, v_2^B) = \alpha v_2^A + v_2^B \end{array}. \tag{14}$$

This payoff specification implicitly assumes that each agent has a lower marginal valuation for the other agent's good. The result also holds in the reverse case, where the payoffs are $\pi_1^{z_1}(v_1^A, v_1^B) = \alpha v_1^A + v_1^B$ and $\pi_2^{z_1}(v_2^A, v_2^B) = v_2^A + \alpha v_2^B$. It also holds when there is no ex-ante preference between the assets, and $\pi_i^{z_1}(v_i^A, v_i^B) = \max\{v_i^A, v_i^B\} + \alpha \min\{v_i^A, v_i^B\}$.

First, we examine whether or not efficiency is feasible when ownership is concentrated, in the sense that one agent owns both assets.

3.1 Concentrated Ownership

The Case of Substitutes: $\alpha < 1$ The first result shows that if, at the status quo, ownership is concentrated, and goods are substitutes, in the sense that $\alpha < 1$, then efficiency is sometimes possible.

This is in contrast to the impossibility result established in Proposition 1 for the case of homogeneous assets.

Proposition 7 *Suppose that* $\alpha < 1$ *and that the status quo is given by allocation* $(AB, 0)$ *(or* $(0, AB)$*). Then, there exist distributions of types* F_1, F_2 *for which it is possible to design feasible and ex-post efficient mechanisms.*

Proof. We look at the case where the status quo allocation is $(AB, 0)$. The case of $(0, AB)$ is analogous. We establish the Proposition by showing that there exist distributions F_1, F_2 with supports $[0,1]^2$ for which efficient trade is possible. In order to apply (10) we first need to investigate what the critical types would be. This task is immediate for agent 2: Since he does not own any of the assets, his outside payoff is 0, so irrespective of his expected payoff at an ex-post efficient assignment, the type vector where the participation constraint binds (the critical type) is $(0, 0)$. Now, since agent 1 owns both assets, his payoff from non-participation depends on his type, and it is given by $v_1^A + \alpha v_1^B$. Along the dimension of asset A, agent 1's non-participation payoff has the highest possible slope, namely 1, and along the dimension of asset B, the slope is α. Therefore, regardless of the shape of the participation payoff determined by an ex-post efficient allocation and the distribution of types, along the dimension of asset A, the critical type for agent 1 is 1, and along the dimension of asset B, it can be any type $v_1^{B*} \in [0, 1]$.

Given these critical types, (11), for the environment under consideration, becomes

$$\begin{aligned}
S(v_1^A, v_1^B, v_2^A, v_2^B) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, v_1^{B*}, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 0) - (1 + \alpha v_1^{B*}) \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + \max\{1 + \alpha v_1^{B*}, 1 + v_2^B, v_1^{B*} + v_2^A, \alpha v_2^A + v \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^A, v_1^B, 0\} - (1 + \alpha v_1^{B*}) \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + \max\{1 + \alpha v_1^{B*}, 1 + v_2^B, v_1^{B*} + v_2^A\} \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^B\} - (1 + \alpha v_1^{B*}).
\end{aligned}$$

We then notice that $S(v_1^A, v_1^B, v_2^A, v_2^B)$ is positive at least in the region where

$$\begin{aligned}
v_1^A + v_2^B &= \max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \\
1 + v_2^B &= \max\{1 + \alpha v_1^{B*}, 1 + v_2^B, v_1^{B*} + v_2^A\} \\
v_1^B &= \max\{v_1^A + \alpha v_1^B, v_1^B, 0\}. \qquad (15)
\end{aligned}$$

Let us call R the region of types where (15) holds. For types in R, $S(v_1^A, v_1^B, v_2^A, v_2^B)$ becomes

$$\begin{aligned}
S(v_1^A, v_1^B, v_2^A, v_2^B) &= -(v_1^A + v_2^B) + 1 + v_2^B + v_1^B - (1 + \alpha v_1^{B*}) \\
&= v_1^B - v_1^A - \alpha v_1^{B*},
\end{aligned}$$

which is strictly positive if

$$v_1^B > v_1^A + \alpha v_1^{B*}. \quad (16)$$

First note that region R is non-empty: After some straightforward simplifications, we get that types in region R satisfy the following equalities:

$$\begin{aligned} v_1^A &\geq \alpha v_2^A \\ v_2^B &\geq \max\{\alpha v_1^B, v_1^B + v_2^A - v_1^A, v_1^{B*} + v_2^A - 1\} \\ v_1^B &\geq v_1^A + \alpha v_1^B. \end{aligned}$$

In the intersection of these regions, we have that

$$\begin{aligned} \min\{v_1^B(1-\alpha), v_1^B - \alpha v_1^{B*}\} &\geq v_1^A \geq \alpha v_2^A \text{ and} \\ v_2^B &\geq \max\{\alpha v_1^B, v_1^B + v_2^A - v_1^A, v_1^{B*} + v_2^A - 1\}. \end{aligned}$$

This region is non-empty; for example, it contains vectors of valuations that satisfy $v_2^A = 0$, $v_2^B = 1$, $v_1^B > v_1^A + \alpha$. Therefore, if the distributions F_1 and F_2 put enough weight on that region, it is possible to design ex-post efficient negotiating procedures. ■

The proof illustrates quite well the forces behind the possibility result: A positive surplus is generated in a region where efficiency dictates that agent 1 keeps asset A and agent 2 obtains asset B. Since agent 2 *marginally* values asset B more than agent 1, it is possible that he pays more than what agent 1 needs to be willing to give away the asset: The payments from each agent in that region are given by

$$\begin{aligned} x_1(v_1, v_2) &= v_2^B - (1 + v_2^B) + (1 + \alpha v_1^{B*}) = \alpha v_1^{B*} \\ x_2(v_1, v_2) &= v_1^A - v_1^B. \end{aligned}$$

Agent 1 is compensated based on his marginal loss at the critical type- that is, αv_1^{B*} while agent 2 pays $v_1^A - v_1^B$. Now, because we are in a region where (16) holds, the sum of these payments is positive, implying that there is a region where surplus is generated. The expected surplus can be positive if the probability that we are in region R is high enough, which is more likely as α decreases. With $\alpha = 1$, we are again in the Myerson-Satterthwaite case, and this region has an empty interior, so the impossibility result reappears.

Another aspect that one can see from the proof of this Proposition is that efficiency requires assets to be heterogeneous: It requires that asset B is valued more by agent 2 than by agent 1- that is $v_2^B \geq \alpha v_1^B -$ and, moreover, that agent 1 likes asset B more than asset A (this is needed in order to satisfy the last inequality (16)). If agent 1 valued the two assets in the same way, then the efficiency region would be empty, which is consistent with the impossibility result we established for the case of homogeneous assets.

It is interesting to note that the above result depends on the initial ownership structure being $(AB, 0)$. If, in the setup given by (14), the ownership structure is either (A, B) or (B, A), efficiency is *never* possible.

This is established next in Proposition 9, and it contradicts the conventional wisdom, which suggests that it is easier to achieve efficiency if property rights are "more balanced," in the sense that both agents own some part of the total endowment.

Now, we look at what happens when ownership is still concentrated, but the assets are complements in the sense that $\alpha > 1$.

The Case of Complements: $\alpha > 1$ If assets are complements, it is not difficult to see that inefficiencies increase if ownership is concentrated- that is, if, at the status quo, one agent owns everything. The strengthening of the conflict (ex-post efficiency indicates that assets are never shared in this case) just gives more force to the impossibility of achieving efficiency. Basically, both players are negotiating over a bundle, and the Myerson-Satterthwaite result applies in full force:

Proposition 8 *Suppose that* $\alpha > 1$ *and that the status quo is given by allocation (AB,0).*[11] *Then, for any distribution of types* F_1, *and* F_2 *with full supports* $[\underline{v}_1^A, \overline{v}_1^A] \times [\underline{v}_1^B, \overline{v}_1^B]$ *and* $[\underline{v}_2^A, \overline{v}_2^A] \times [\underline{v}_2^B, \overline{v}_2^B]$, *respectively, that satisfy* $0 < \underline{v}_i^j < \overline{v}_i^j < \infty$,

$$\underline{v}_2^B < \alpha \overline{v}_1^B \tag{17}$$

and

$$\underline{v}_2^A < \overline{v}_1^A \tag{18}$$

there is no feasible and ex-post efficient mechanism.

The proof of Proposition 8 amounts to examining the sign of S over all regions of valuations and, thereby, establishing that it is negative. Since it is quite lengthy, we relegate it to Appendix A.

Proposition 8 says that if one agent owns two heterogenous assets that are complementary, then when gains from trade are uncertain (conditions (17) and (18) guarantee this), it is impossible to have ex-post efficient trade. Now, we turn to the case where ownership is dispersed, in the sense that each agent owns one asset. There, we see that the results of this section are reversed.

3.2 Dispersed Ownership

The Case of Substitutes: $\alpha < 1$ The first result here is that when assets are substitutes and ownership is dispersed, efficiency is impossible.

Proposition 9 *Suppose that* $\alpha < 1$ *and that the status quo is given by allocation (A,B). Then for any distribution of types* F_1, *and* F_2 *with full supports* $[\underline{v}_1^A, \overline{v}_1^A] \times [\underline{v}_1^B, \overline{v}_1^B]$ *and* $[\underline{v}_2^A, \overline{v}_2^A] \times [\underline{v}_2^B, \overline{v}_2^B]$, *respectively, where* $0 < \underline{v}_i^j < \overline{v}_i^j < \infty$ *that satisfy*

$$\underline{v}_1^B \leq \overline{v}_2^B \tag{19}$$

[11] The case where agent 2 owns both assets $(0, AB)$ is a simple relabeling.

and

$$\underline{v}_2^A \leq \overline{v}_1^A \tag{20}$$

there is no feasible and ex-post efficient mechanism.

Proposition 9 establishes that when assets are substitutes, in the sense that $\alpha < 1$, and each agent owns one asset, it is impossible to have ex-post efficient trade when gains from trade are uncertain (conditions (19) and (20)). The same forces that made efficiency possible when ownership rights were concentrated (Proposition 7) go against efficiency when ownership is dispersed. An agent acquiring an asset must compensate the owner based on his marginal disutility of losing it, which is 1. But at the same time, acquiring it either has a lower marginal utility α, or it creates a lower marginal utility for the other issue. Combined, these effects make it impossible to adequately compensate the owner of an asset. Its proof its analogous to that of Proposition 8 and can be found in Appendix A.

We now turn to the case of complementary assets.

The Case of Complements: $\alpha > 1$ In the case of complementary assets, dispersed ownership makes the existence of efficient mechanisms sometimes possible. The marginal valuation of an asset by a potential buyer is bigger than the one of the potential seller since it "completes" a bundle. This can generate a surplus, as we see in the next Proposition:

Proposition 10 *Suppose that $\alpha > 1$ and that the status quo is given by (A, B) or (B, A). Then there exist distributions of types F_1, F_2 for which a feasible and ex-post efficient mechanism exists.*

Proof. We establish the Proposition by showing that there exist distributions F_1, F_2 with supports $[0,1]^2$ for which efficient trade is possible. Given status quo (A, B), it is easy to see that the critical types for agent 1 and 2 are given by $(v_1^A, v_1^B) = (1, 0)$ and $(v_2^A, v_2^B) = (0, 1)$. The expression (10) for each v_1, v_2 becomes:

$$\begin{aligned}
S(v_1, v_2) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, 0, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 1) - 2 \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \\
&\quad + \max\{1, 1 + v_2^B, v_2^A, \alpha v_2^A + v_2^B\} \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^A + 1, v_1^B, 1\} - 2,
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
S(v_1, v_2) &= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \\
&\quad + \max\{1 + v_2^B, \alpha v_2^A + v_2^B\} + \max\{v_1^A + \alpha v_1^B, v_1^A + 1\} - 2.
\end{aligned}$$

Suppose that $v_1^B = v_2^A = 1$; $v_1^A > v_2^B$ and $v_1^A > 2 - a$ and $v_2^B = 2 - \alpha$. It is straightforward to check that $S(v_1, v_2)$ is strictly positive. Then, if distributions F_1, F_2 put enough mass on this region, efficiency is possible. ■

The critical force that makes efficiency possible is that the potential buyer of an asset has a higher marginal valuation for it than the potential seller does. As can be seen from the proof of Proposition 10, this force increases as α increases: The region where the surplus is positive increases, too.

In Proposition 7, we saw that efficiency can be feasible when one agent exclusively owns both assets that are substitutes. In Proposition 9, we saw that when each agent owns an asset and the assets are substitutes, efficiency is not feasible. When assets are complements, the reverse is true, as we saw in Propositions 8 and 10. These results show that in determining which assets to negotiate about simultaneously, one has to think about the nature of the assets (the α in our model), as well as the initial ownership structure.

This completes our investigation of the conditions under which multi-asset negotiations can achieve an efficient outcome in the presence of asymmetric information when the ownership of each asset is exclusive. In the following section, we investigate the role of initial ownership structures when agents co-own assets.

4. Trading Mechanisms with Co-ownership (Partnerships)

A partnership consists of a group of individuals that co-own a number of assets. We examine which ownership structures make efficiency possible and ask why they do so, and what relationship between the degree of asymmetries across partners and the ownership structures make efficient dissolution possible. In order to focus on the ownership issue, we abstract from substitutabilities and complementarities. We first examine partnerships of one asset. Our results in this section will shed new light on the impossibility and possibility results of Myerson and Satterthwaite (1983) and Cramton, Gibbons and Klemperer (1987), respectively.

Co-ownership of One Asset The seminal paper by Cramton, Gibbons and Klemperer (1987) examines partnerships where agents' valuations are drawn from the same distribution and their valuations are given by $\pi_i(v_i) = v_i$. In that context, they show that when the shares are close to $r_i = \frac{1}{I}$, it is possible to find a mechanism that is feasible and ex-post efficient. Comparing this result to the one in Myerson and Satterthwaite (1983) may lead to the conclusion that what makes the existence of ex-post efficient mechanisms possible is the *equality* in property rights. Here, we allow for asymmetric type distributions and more-general payoff functions, and we find that feasible and ex-post efficient mechanisms exist if all agents' valuations from the asset evaluated at the vector of critical types are equal.

The Setup Agent i's payoff from owning a fraction r of the asset is $r \cdot \pi_i(v_i)$, where π_i is increasing and convex in v_i, and it is zero otherwise. The partnership is characterized by the initial property rights, $Q = (r_1, ..., r_I)$ with $\Sigma_{i \in I} r_i = 1$. Agent i's payoff at the status quo $Q = (r_1, ..., r_I)$, is $\underline{U}_i(v_i) = r_i \cdot \pi_i(v_i)$.

An ex-post efficient assignment p^* is one where for each v, the agent with the highest $\pi_i(v_i)$ is awarded exclusive ownership of the asset. Then, the expected payoff for agent i, when his valuation is v_i at the ex-post efficient assignment, is

$$U_i(v_i, v_i; (p^*, x)) = \prod_{\substack{j \in I \\ j \neq i}} F_j(\pi_j^{-1}(\pi_i(v_i))) \cdot \pi_i(v_i) + E_{v_{-i}}[x_i(v)],$$

where $\prod_{\substack{j \in I \\ j \neq i}} F_j(\pi_j^{-1}(\pi_i(v_i)))$ denotes the probability that i's payoff for the asset is the highest, given that his valuation is v_i.

Our first result sheds light on the impossibility result in Myerson and Satterthwaite (1983) and the possibility result in Cramton, Gibbons and Klemperer (1987). We solve for the ownership structure that minimizes the sum of K_i^*'s, where each K_i^* is given by (7), and show that at the optimal one $\pi_i(v_i^*) = \pi_j(v_j^*)$ for all $i, j \in I$. This is because the marginal transfer that must be paid to an agent i if his property rights are increased, is his valuation at the critical type $\pi_i(v_i^*)$. It is "cheap" to increase the property rights of a player where the valuation at the critical type $\pi_i(v_i^*)$ is low, and a lot can be saved from reducing the property rights of a player with high $\pi_i(v_i^*)$. This implies that an environment that has the best shot at efficiency is one where all agents' payoffs at the critical type are equal. As we later see, in such an environment, the property rights can be extremely unequal.

For any agent i, consider $K_i(r_i) = \max_{v_i}[r_i \pi_i(v_i) - E_{v_{-i}} W(v_i, v_{-i})]$. From (7), we know that the maximizer is v_i^* and the maximized value is $K_i(r_i)$. It follows that this is the smallest constant that must be added to the standard VCG mechanism such that the participation constraints are satisfied.

Lemma 1 *Suppose that the ownership share of agent i is marginally increased. Then, the marginal increase in the transfers necessary to guarantee voluntary participation is*

$$\frac{dK_i(r_i)}{dr_i} = \pi_i(v_i^*).$$

Proof. The marginal increase in the transfers necessary to guarantee voluntary participation is

$$\begin{aligned} \frac{dK_i(r_i)}{dr_i} &= \pi_i(v_i^*) + r_i \frac{\partial v_i^*}{\partial r_i} - \frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i} \frac{\partial v_i^*}{\partial r_i} \\ &= \pi_i(v_i^*) - \frac{\partial v_i^*}{\partial r_i} \cdot \left(\frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i} - r_i \frac{\partial \pi_i(v_i^*)}{\partial v_i} \right) \\ &= \pi_i(v_i^*), \text{ for all } i \end{aligned}$$

where the last inequality comes just from the fact that at an (interior) critical type, the envelope theorem allows us to conclude that $\frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i} - r_i \frac{\partial \pi_i(v_i^*)}{\partial v_i} = 0$. ■

Lemma 2 *Consider a partnership and suppose that π_i is strictly increasing in v_i, for all agents. If all valuation functions have the same range ($\pi_i(\underline{v}_i) = \underline{\pi}$, and $\pi_i(\bar{v}_i) = \bar{\pi}$), then, the transfer-minimizing property rights $(r_1, ..., r_I)$ are such that $\pi_i(v_i^*) = \pi_j(v_j^*)$ for all $i, j \in I$. Moreover, such property rights exist.*

Proof. Suppose that we can choose $r_i's$ so as to minimize the sum of transfers necessary to guarantee the agents' voluntary participation, in other words, we seek the ownership structure that solves

$$\min_{r_i, i=1,..I} \Sigma_{i\in I} K_i^*(r_i) \text{ subject to } \Sigma_{i\in I} r_i = 1. \tag{21}$$

The first-order conditions of this minimization problem imply $\frac{dK_i(r_i)}{dr_i} = \frac{dK_j(r_j)}{dr_j}$, which from Lemma 1 becomes $\pi_i(v_i^*) = \pi_j(v_j^*)$.

Now, we establish that for any distributions F_i, $i \in I$, there exists an initial ownership structure $(r_1, ..., r_I)$ that guarantees that $\pi_i(v_i^*) = \pi_j(v_j^*)$ holds.

First, note that at interior critical types, participation and non-participation payoffs must be tangent, implying that

$$\prod_{\substack{j\in I \\ j\neq i}} F_j(\pi_j^{-1}(\pi_i(v_i^*)))\pi_i'(v_i^*) = r_i \cdot \pi_i'(v_i^*). \tag{22}$$

Defining $G_i(s) = \prod_{\substack{j\in I \\ j\neq i}} (F_j \circ \pi_j^{-1})(s)$, and noticing that it is invertible (since it is increasing), (22) can be rewritten as

$$\pi_i(v_i^*) = G_i^{-1}(r_i). \tag{23}$$

Therefore, for $\pi_i(v_i^*) = \pi_j(v_j^*)$ to be true, we must have

$$G_i(G_j^{-1}(r_j)) = r_i \text{ for all } i, \tag{24}$$

which, for a given r_i determines r_j. Now, we only have to check the consistency requirement that $\sum_{1=1}^{I} r_i = 1$, so, for $i = 1$ and $j = I$, (24) becomes

$$G_1^{-1}(r_1) - G_I^{-1}(1 - r_1 - G_2(G_1^{-1}(r_1)) - ... - G_{I-1}(G_1^{-1}(r_1))) = 0. \tag{25}$$

Noticing that $G_1^{-1}(0) - G_I^{-1}(1) = -1$ and $G_1^{-1}(1) - G_I^{-1}(0) = 1$, continuity of G_1^{-1} implies the existence of $r_1 \in (0, 1)$ such that (25) holds. ■

In a symmetric linear environment (like the one in Cramton, Gibbons and Klemperer (1987)), equality of payoffs at the critical types is equivalent to having all property rights r_i equalized. This is one way to explain what is going on behind the possibility result of Cramton, Gibbons and Klemperer (1987): In a symmetric environment, equal property rights maximize the expected surplus of a mechanism by equating critical types.[12] On the other hand, extreme property rights, as in Myerson and Satterthwaite (1983), imply that the critical type for the seller is his highest valuation, while the critical type for the buyer is his lowest valuation, thus maximizing the difference in agents' payoffs at their critical types.

From Lemma 2, it follows directly that in order to design efficiently dissolvable partnerships, one should aim for environments where the valuations of critical types are the same across agents. Our next Proposition shows that when this is true, efficiency is indeed feasible.

Proposition 11 *Consider a partnership. If property rights* $(r_1, ..., r_I)$ *are such that*

$$\pi_i(v_i^*) = \pi_j(v_j^*), \text{ for all } i \text{ and } j, \tag{26}$$

then there exists a feasible and ex-post efficient mechanism.

Proof. [13]From (10), it is possible to design an ex-post efficient mechanism iff

$$E\left[-(I-1)\max_{i\in I}\{\pi_i(v_i)\} + \sum_{i\in I}\max\{\pi_i(v_i^*), \pi_{-i}(v_{-i})\}\right] \geq \sum_{i\in I} r_i \cdot \pi_i(v_i^*). \tag{27}$$

Therefore, a sufficient condition for (27) is that

$$-(I-1)\max_{i\in I}\{\pi_i(v_i)\} + \sum_{i\in I}\max\{\pi_i(v_i^*), \pi_{-i}(v_{-i})\} \geq \sum_{i\in I} r_i \cdot \pi_i(v_i^*). \tag{28}$$

We just need to verify that (28) is satisfied whenever (26) is satisfied.

Suppose that, for some vector of valuations v_i, we have that

$$\pi_k(v_k) = \max_{i\in I}\{\pi_i(v_i)\}.$$

Then, for this vector of valuations, (28) reduces to

$$-(I-1)\pi_k(v_k) + (I-1)\pi_k(v_k) + \max\{\pi_k(v_k^*), \pi_{-k}(v_{-k})\} \geq \sum_{i\in I} r_i\pi_i(v_i^*),$$

which is always true. Since this holds for any v, (28) always holds. ■

Proposition 11 is related to Proposition 2 in Schweizer (2006), which states that if the social surplus is a linear function of the information profile and a convex function of the collective decision, there exists a default option such that efficient negotiations are possible. In relation to Schweizer (2006), we not only state that there exist property rights that guarantee the efficient dissolution of asymmetric partnership,

[12] In Lemma 2, we considered the case in which the range of valuation functions is the same. When this is not true, we may be at a 'corner' solution where the marginal costs of participation of each agent are not equal.

[13] The proof of this Proposition is much shorter than our original proof thanks to Y-K Che.

but we also identify that the critical condition is that $\pi_i(v_i^*) = \pi_j(v_j^*)$, which allows us to infer exactly how the properties rights should be. For linear environments, where the condition reduces to $v_i^* = v_j^*$, the result was discovered independently by Che (2006). In relation to this work, we not only identify the role of this condition in greater generality, but, more importantly we discover (cf. Lemma 1) why it is crucial: An agent's payoff at the critical type is the marginal cost of increasing his ownership share while maintaining voluntary participation.

For the linear case where $\pi_i(v_i) = v_i$, (26) reduces to $v_i^* = v_j^*$. If, moreover, all types are distributed according to the same distribution F, we have that $v_i^* = v_j^*$ is satisfied if $r_i \equiv \frac{1}{I}$, exactly as in Cramton, Gibbons and Klemperer (1987). However, when distributions are asymmetric, the property rights that guarantee the condition in Proposition 11 can be extremely unequal:

Example 1 *Consider a partnership with two agents, $\pi_i(v_i) = v_i$, $F_1(v_1) = v_1^n$ and $F_2(v_2) = v_2^{\frac{1}{n}}$. Then, it is easy to see that $v_1^* = F_2^{-1}(r_1) = r_1^n$ and $v_2^* = F_1^{-1}(r_2) = r_2^{\frac{1}{n}}$. From Proposition 11, we know that if $v_1^* = v_2^*$, efficient dissolution is possible. For this example, this condition is equivalent to*

$$r_1^n = r_2^{\frac{1}{n}}. \tag{29}$$

Recalling that $r_1 + r_2 = 1$, (29) reduces to

$$r_1^n = (1 - r_1)^{\frac{1}{n}}.$$

For $n = 3$ we obtain $r_1 = 0.8243$ and $r_2 = 0.1757$, which give us that $v_1^* = v_2^* = 0.56009$. Moreover, a simple calculation shows that for these distributions, with property rights of $r_i = \frac{1}{2}$, there is no possibility of efficient dissolution. For $n = 99$, optimal property rights are even more extreme: $r_1 = 0.99926$ and $r_2 = 0.00074$. For this case, the corresponding critical types are $v_1^* = v_2^* = 0.92933$.

The previous example shows that, for certain distributions, very extreme property rights are needed in order to have efficient dissolution of the partnership, quite contrary to the intuition one gets from the discussion of symmetric environments. In fact, for the very extreme case of $n = 99$, property rights very close to $(1, 0)$ are needed. It is useful to contrast this result with the one in Myerson and Satterthwaite (1983), which shows that $(r_1, r_2) = (1, 0)$ will never allow an efficient dissolution. In fact, our example shows that arbitrarily close-to-extreme property rights can be needed for efficient dissolution, but property rights that *are* extreme will never allow it. Moreover, this example suggests that agent 1, whose valuation is more likely to be higher, must own a higher proportion of the asset and vice versa. This turns out to be a general result with interesting economic consequences:

Corollary 1 *Let us suppose that*

$$F_1 \circ \pi_1^{-1}(\cdot) \geq F_2 \circ \pi_2^{-1}(\cdot) \geq ... \geq F_I \circ \pi_I^{-1}(\cdot).^{14} \tag{30}$$

[14] The conditions in (30) are equivalent to the distributions of valuations being ordered according to FOSD (first-order stochastic dominance) since $F_i \circ \pi_i^{-1}(x) = P(\pi_i(v_i) \leq x)$.

Then, the property rights that guarantee the possibility of efficient dissolution satisfy $r_1 \leq r_2 \leq ...r_I$.

Proof. We know that at the critical types $\prod_{j \neq i} F_j(\pi_j^{-1}(\pi_i(v_i^*))) = r_i$. Moreover, for the property rights that guarantee dissolution, we have $\pi(v_i^*) \equiv x$ for all $i \in I$. Therefore, we have $r_i = \prod_{j \neq i} [F_j \circ \pi_j^{-1}](x) = r_i$, from which we obtain that $\frac{r_i}{r_j} = \frac{[F_j \circ \pi_j^{-1}](x)}{[F_i \circ \pi_i^{-1}](x)} > 1$. ■

Corollary 1 has important implications for many situations:

Professional Players: In the case of Luis Jiménez's negotiation mentioned earlier, what might have contributed to the breakdown of negotiations, is the symmetry of the property rights. Fiorentina, which had a much higher probability of high valuations (playing in Series A, there is more at stake), owned as many shares as Ternana, which had a much smaller probability of high valuations. An asymmetric partnership would have been advisable, with Fiorentina owning a significantly higher number of shares.

Joint Ventures: The same could be said, in general, about other joint ventures. Even if the participation of all associates is needed, Corollary 1 suggests that the ones likely to benefit the most ex-post (for example, the ones with more marketing muscle) should own bigger shares of the project. If this fails, it may be impossible to find a mechanism that transfers the control efficiently to one of the participants.

R&D and Firm Structure: It is well known that in many cases, firms that are better at generating an invention are not necessarily the best ones to develop applications for it. In the introduction, we mentioned the failed negotiation for the technology used in BlackBerries. Our result hints at an explanation: To have efficient trade, firms that are better at developing applications (F_i biased toward high values of v_i) should also have high probabilities of winning the research race (higher r_i). It also provides a rationale for the integration of research and production within one firm: When independent laboratories conduct research, inventions have to be sold in order to be developed. The asymmetric information present in these transactions may lead to a breakdown in negotiations, leading to a slower implementation of the new technologies and forfeited profits.

We continue by noting that our findings generalize straightforwardly to the case where a partnership owns multiple assets: firms co-owning multiple plants, patents and copyrights, or a married couple that seeks a divorce and owns multiple properties.

Co-ownership of Multiple Assets Here, we look at the case where there are I individuals who jointly own K assets. Agent i's valuation of asset k is given by $\pi_{ik}(v_i^k)$, and his valuation of a subset of assets $S \in K$ is additive: $\sum_{k \in S} \pi_{ik}(v_i^k)$ (so, assets are neither complements nor substitutes). The ownership share of asset k by agent i is denoted by r_i^k; if there is disagreement his payoff is given by $\sum_{k \in K} r_i^k \cdot \pi_{ik}(v_i^k)$.

In this context, exactly as in the case of single-asset partnerships, one can find property rights that guarantee the existence of efficient mechanisms.

Corollary 2 *Consider a multi-asset partnership where assets are neither complements nor substitutes. Then,*

- *If the property rights are such that* $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_j^{k*})$ *for all* $i, j \in I, k \in K$, *there exists a feasible and ex-post efficient mechanism.*
- *If* π_{ik} *is strictly increasing in* v_i^k, *for all* i *and* k, *and all valuation functions have the same range, then it is always possible to find property rights such that* $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_j^{k*})$ *for all* $i, j \in I, k \in K$.
- *If for every asset* k *there exists an agent* i *such that* $r_{ik} = 1$, *then there is no feasible and ex-post efficient mechanism.*

The result in this section shows that multi-asset partnerships with additive-separable payoffs across assets, can be dissolved efficiently if the initial property rights are such that all agents' payoffs at the critical types are equal across all assets and all agents. This is for exactly the same reasons, as the possibility result that we established for single-asset partnerships.

Partnerships have an interesting structure: The co-ownership provides a link across agents that allows one to think about the sum of payments in the program (21), which is not possible in the cases of exclusive ownership that we examined earlier. Still, this result highlights the important role of critical types, which is also present (together with other forces) ,in situations where ownership is exclusive as we saw earlier.

5. Conclusions

In this paper, we investigated the ownership structures under which the presence of complementarities and substitutabilities among assets for trade help alleviate the inefficiencies that arise from asymmetric information.

First, we analyzed the case where two agents negotiate over multiple homogeneous assets that they perceive either as substitutes or as complements. When assets are homogeneous, private information is one-dimensional. Then, when ownership is concentrated, in the sense that one agent owns all assets, we showed that efficiency is never possible regardless of whether the goods are complements or substitutes. We also showed that when assets are complements, simultaneous negotiation exacerbates the conflict: The subsidy needed to achieve efficiency is increasing in the degree of complementarity between the two assets. When assets are substitutes, the effect of substitutability is ambiguous: As assets becomes more substitutable, this increases the surplus by reducing the seller's outside option on the one hand, but it decreases the buyer's payments on the other hand. When ownership is dispersed, in the sense that each asset is owned by a different agent, efficiency is often possible. There, the surplus is increasing in the degree of complementarity of the assets. The reason is that the higher the degree of complementarity, the higher the difference in marginal utilities between the agent acquiring the second unit and the agent selling away his asset. Again, when assets are substitutes, the effect of higher substitutability is ambiguous. As substitutability drops, trade opportunities emerge; however, payments generated may not be high enough to cover the outside options.

Our analysis suggests that when assets are homogeneous, the results echo the pre-existing impossibility (à la Myerson-Satterwhaite (1983)) and possibility (à la Cramton, Gibbons and Klemperer (1987)) results. This is no longer the case when assets are heterogeneous. There, private information is multi-dimensional, and this has an important effect on the inefficiencies created due to information rents. In some cases, where we have impossibility with single-dimensional private information, we obtain efficiency in the multidimensional setting: For example, efficiency can be feasible for heterogeneous assets that are substitutes and are all owned by the same agent.

Our findings suggest that if agents have a choice of which issues to put on the table in order to make efficiency more likely, they would have to consider how similar these issues are. For heterogeneous issues, if one agent has control of all the assets (issues), he should put more issues that exhibit substitutabilities; if assets are controlled by different agents, more issues that exhibit complementarities should be put on the table.

We then investigated the role of initial property rights in situations where co-ownership is allowed. We showed that in order to guarantee the existence of ex-post efficient mechanisms, property rights should be biased towards the agents with a higher probability of having a high valuation of the asset. Finally, we used these results to discuss the breakdown in negotiations where initial ownership is well-balanced, and, based on previous work, one would have expected ex-post efficient trade to be possible. We also discussed implications for the design of joint ventures and research partnerships.

The main lesson of our analysis is that in the presence of asymmetric information, negotiations over multiple issues cannot generally be viewed as a union of single-issue negotiations. If the objective is to design a negotiation procedure that maximizes gains from trade, one has to think carefully about what issues to put together on the table. The relevant variables are the issues' nature and the initial control of rights. We hope that our findings provide some guidelines for how one should go about doing so.

6. Appendix A: Proofs

Proof of Proposition 1

We establish the result for the case that the status quo allocation is $(2, 0)$. First, note that, given this status quo, it is easy to see that, regardless of the distributions of valuations, and of the exact form of the ex-post efficient assignment, the critical types for agent 1 and 2, are always given by $v_1^* = \bar{v}_1$ and $v_2^* = \underline{v}_2$, respectively. This is because the slope of the seller's payoff from non-participation is $1 + \alpha$, while the slope of his payoff from an ex-post efficient assignment is weakly less than $1 + \alpha$, for all $v_1 \in [\underline{v}_1, \bar{v}_1]$. Now, the slope of the buyer's non-participation payoff is 0, and the slope of his payoff from an ex-post efficient assignment is weakly greater than 0, for all $v_2 \in [\underline{v}_2, \bar{v}_2]$.

With these critical types, $S(v_1, v_2, \alpha)$ from (10) becomes

$$\begin{aligned} S(v_1, v_2, \alpha) &= -W(v_1, v_2) + W(\bar{v}_1, v_2) + W(v_1, \underline{v}_2) - (1+\alpha)\bar{v}_1 \\ &= -\max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\} \\ &\quad +\max\{(1+\alpha)\bar{v}_1, \bar{v}_1+v_2, (1+\alpha)v_2\} \\ &\quad +\max\{(1+\alpha)v_1, v_1+\underline{v}_2, (1+\alpha)\underline{v}_2\} - (1+\alpha)\bar{v}_1. \end{aligned} \tag{31}$$

Notice that we explicitly note the dependence of the surplus on the degree of complementarity/substitutability of the goods α. In order to determine the sign of S, we examine the following scenarios:

If $(1+\alpha)\bar{v}_1 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1+v_2, (1+\alpha)v_2\}$, then the second and the last terms on the right-hand-side of (31) cancel out, and we immediately have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -\max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\} \\ &\quad +\max\{(1+\alpha)v_1, v_1+\underline{v}_2, (1+\alpha)\underline{v}_2\} \leq 0. \end{aligned}$$

If $(1+\alpha)v_2 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1+v_2, (1+\alpha)v_2\}$, then it follows that $(1+\alpha)v_2 = \max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\}$, which implies that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -(1+\alpha)\bar{v}_1 + \max\{(1+\alpha)v_1, v_1+\underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{(1+\alpha)(v_1-\bar{v}_1), (v_1-\bar{v}_1)+(\underline{v}_2-\alpha\bar{v}_1), (1+\alpha)(\underline{v}_2-\bar{v}_1)\} \leq 0, \end{aligned}$$

where the last inequality follows from the fact that $\underline{v}_2 < \alpha\bar{v}_1$.

Finally, if $\bar{v}_1 + v_2 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1+v_2, (1+\alpha)v_2\}$, this implies that $v_2 > \alpha\bar{v}_1$, so $\max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\}$ is either v_1+v_2 if $v_1 > \alpha v_2$, or $(1+\alpha)v_2$ otherwise. If $v_1 > \alpha v_2$, then $v_1+v_2 = \max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -v_1 - \alpha\bar{v}_1 + \max\{(1+\alpha)v_1, v_1+\underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{\alpha(v_1-\bar{v}_1), \underline{v}_2-\alpha\bar{v}_1, (\underline{v}_2-\alpha\bar{v}_1)+(\alpha\underline{v}_2-v_1)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 > \underline{v}_2$, and the fact that, in this case, $v_1 \geq \alpha v_2$. If $v_1 < \alpha v_2$, then $(1+\alpha)v_2 = \max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -\alpha v_2 - \alpha\bar{v}_1 + \max\{(1+\alpha)v_1, v_1+\underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{(v_1-\alpha v_2)+\alpha(v_1-\bar{v}_1), (v_1-\alpha v_2)+(\underline{v}_2-\alpha\bar{v}_1), (\underline{v}_2-\alpha\bar{v}_1)+\alpha(\underline{v}_2-v_2)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 > \underline{v}_2$, and the fact that, in this case, $v_1 \leq \alpha v_2$.

Therefore, $S(v_1, v_2, \alpha)$ is less than zero for all v_1, v_2. Moreover, there are regions of types with a non-empty interior where $S(v_1, v_2, \alpha) < 0$. These two observations together establish that it is impossible to design ex-post efficient mechanisms. ■

Proof of Proposition 3

We start by writing down (10) for the case under consideration.

$$\begin{aligned} S(v,\alpha) \quad = \quad & -\max\{(1+\alpha)v_1,(1+\alpha)v_2\} \\ & +\max\{(1+\alpha)v_1^*,(1+\alpha)v_2\} \\ & +\max\{(1+\alpha)v_2^*,(1+\alpha)v_1\} - v_1^* - v_2^*. \end{aligned} \tag{32}$$

This can be rewritten as

$$S(v,\alpha) = \begin{cases} (1+\alpha)(v_2-v_1)+\alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 > v_2 \\ (1+\alpha)v_2 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 > v_2 \\ \alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 > v_2 \\ -(1+\alpha)v_1 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 > v_2 \\ \alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 < v_2 \\ (1+\alpha)v_1 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 < v_2 \\ (1+\alpha)(v_1-v_2)+\alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 < v_2 \\ -(1+\alpha)v_2 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 < v_2 \end{cases}. \tag{33}$$

Consider the situation where $v_1^* \geq v_2^*$. Then, the first case is impossible, and the second, third and fourth yield a nonnegative surplus. Moreover, if $\alpha v_2^* \geq v_1^*$, the fifth to eighth cases are also nonnegative. ■

Proof of Proposition 4

We want to show that $\bar{S}(\alpha)$ in 10 is increasing in a. First, we calculate the surplus for a given v_1 and then we integrate with respect of v_1 in order to get $\bar{S}(\alpha)$. In these calculations, we use the observation that at the critical type of agent 1, it must hold that $v_1^* = F^{-1}(\frac{1}{1+\alpha})$.

Now, for $v_1 \geq v_1^*$ we have that

$$\begin{aligned} S(v_1) \quad &= \quad \int_{\underline{v}}^{v^*} (\alpha-1)v^* dF(v_2) + \int_{v^*}^{v_1} [(1+\alpha)v_2 - 2v^*]dF(v_2) + \int_{v_1}^{\overline{v}} [(1+\alpha)v_1 - 2v^*]dF(v_2) \\ &= \quad (\alpha-1)v^*F(v^*) - 2v^*(1-F(v^*)) + (1+\alpha)[\int_{v^*}^{v_1} v_2 dF(v_2) + v_1(1-F(v_1))] \\ &= \quad -v^* + (1+\alpha)\int_{v^*}^{v_1} v_2 dF(v_2) + (1+\alpha)v_1(1-F(v_1)) \end{aligned}$$

and for $v_1 \leq v_1^*$ we have that

$$
\begin{aligned}
S(v_1) &= \int_{\underline{v}}^{v_1} [2\alpha v^* - (1+\alpha)v_1] dF(v_2) + \int_{v_1}^{v^*} [2\alpha v^* - (1+\alpha)v_2] dF(v_2) + \int_{v^*}^{\overline{v}} (\alpha - 1)v^* dF(v_2) \\
&= 2\alpha v^* F(v^*) - (1+\alpha)v_1 F(v_1) - \int_{v_1}^{v^*} (1+\alpha)v_2 dF(v_2) + (\alpha-1)v^*(1 - F(v^*)) \\
&= (1+\alpha)v^* F(v^*) - (1+\alpha)[v_1 F(v_1) + \int_{v_1}^{v^*} v_2 dF(v_2)] + (\alpha - 1)v^* \\
&= \alpha v^* - (1+\alpha)v_1 F(v_1) - (1+\alpha) \int_{v_1}^{v^*} v_2 dF(v_2).
\end{aligned}
$$

Using the two expressions derived previously, we get that $\bar{S} = \int S(v_1)dv_1$ can be written as

$$
\begin{aligned}
\bar{S} &= \alpha v^* F(v^*) - v^*(1 - F(v^*)) \\
&\quad -(1+\alpha) \int_{\underline{v}}^{v^*} vF(v)dF(v) + (1+\alpha) \int_{v^*}^{\overline{v}} v(1 - F(v))dF(v) \\
&\quad -(1+\alpha) \int_{\underline{v}}^{v^*} \int_{v_1}^{v^*} v_2 dF(v_2)dF(v_1) + (1+\alpha) \int_{v^*}^{\overline{v}} \int_{v^*}^{v_1} v_2 dF(v_2)dF(v_1)
\end{aligned}
$$

Noting that the terms in the first line cancel out, and using integration by parts for the terms in the third line, we get

$$
\bar{S} = -2(1+\alpha) \int_{\underline{v}}^{v^*} vF(v)dF(v) + 2(1+\alpha) \int_{v^*}^{\overline{v}} v(1 - F(v))dF(v).
$$

We can then write

$$
\begin{aligned}
\frac{1}{2}\frac{\partial \bar{S}}{\partial \alpha} &= -\int_{\underline{v}}^{v^*} vF(v)dF(v) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) \\
&\quad -(1+\alpha)v^*F(v^*)dF(v^*)\frac{\partial v^*}{\partial \alpha} - (1+\alpha)v^*(1-F(v^*))dF(v^*)\frac{\partial v^*}{\partial \alpha} \\
&= -\int_{\underline{v}}^{v^*} vF(v)dF(v) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) + v^*F(v^*) \\
&\geq -v^*F(v^*) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) + v^*F(v^*) \\
&= \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) \\
&\geq 0,
\end{aligned}
$$

from which we can conclude that the expected surplus is increasing in the degree of complementarity of assets α. ■

Proof of Proposition 8

The surplus equals:

$$
\begin{aligned}
S(v_1^A, v_1^B, v_2^A, v_2^B) &= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \text{ (A)} \\
&\quad + \max\{\overline{v}_1^A + \alpha \overline{v}_1^B, \overline{v}_1^A + v_2^B, \overline{v}_1^B + v_2^A, \alpha v_2^A + v_2^B\} \text{ (B)} \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^A + \underline{v}_2^B, v_1^B + \underline{v}_2^A, \alpha \underline{v}_2^A + \underline{v}_2^B\} \text{ (C)} \\
&\quad -\overline{v}_1^A - \alpha \overline{v}_1^B
\end{aligned}
$$

We now establish that under (17) and (18), the surplus S is negative: In order to do that, we have to examine a number of straightforward cases:

Case 1: $A = v_1^A + \alpha v_1^B$

In this case, C must be $v_1^A + \alpha v_1^B$, and, because $\alpha > 1$, $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. Then, the surplus equals to $S = 0$.

Case 2: $A = v_1^A + v_2^B$

This case implies that the following inequalities must be true:

$$
\begin{aligned}
v_1^A + v_2^B &\geq v_1^A + \alpha v_1^B \text{ } (2.i) \\
&\geq v_1^B + v_2^A \text{ } (2.ii) \\
&\geq \alpha v_2^A + v_2^B \text{ } (2.iii)
\end{aligned}
$$

With this information, we immediately have that $\overline{v}_1^A + v_2^B \geq \alpha v_2^A + v_2^B$ and $v_1^A + \underline{v}_2^B \geq \alpha \underline{v}_2^A + \underline{v}_2^B$. So, we have the following cases to consider:

Case 2.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. The surplus can be written as $S = -v_1^A - v_2^B + C = C - A$. Thus, S is always negative (this is implied by $2.i$, $2.ii$ and $2.iii$).

Case 2.1: $B = \overline{v}_1^A + v_2^B$. The surplus can be written as $S = -v_1^A - \alpha \overline{v}_1^B + C$. First, if $C = v_1^A + \alpha v_1^B$, then we have that $S = \alpha(v_1^B - \overline{v}_1^B) \leq 0$. If $C = v_1^A + \underline{v}_2^B$,then $S = \underline{v}_2^B - \alpha \overline{v}_1^B$, which is negative because of (18). And if $C = v_1^B + \underline{v}_2^A$, then $S = v_1^B - \alpha \overline{v}_1^B + \underline{v}_2^A - v_1^A$, which because of $2.iii$ becomes $S \leq v_1^B - \alpha \overline{v}_1^B < 0$.

Case 2.3: $B = \overline{v}_1^B + v_2^A$. The surplus can be then written as $S = -v_1^A - v_2^B + \overline{v}_1^B + v_2^A - \overline{v}_1^A - \alpha \overline{v}_1^B + C$. First, if $C = v_1^A + \alpha v_1^B$, $S = -v_1^A - v_2^B + v_1^A + \alpha v_1^B + \overline{v}_1^B + v_2^A - \overline{v}_1^A - \alpha \overline{v}_1^B \leq \overline{v}_1^B - \alpha \overline{v}_1^B + v_2^A - \overline{v}_1^A \leq \overline{v}_1^B - \alpha \overline{v}_1^B$. For the first inequality we used $2.i$, and for the second $2.iii$. If $C = v_1^A + \underline{v}_2^B$,then $S = v_2^A - \overline{v}_1^A + \overline{v}_1^B(1 - \alpha) + \underline{v}_2^B - v_2^B$. It is easy to see, using iii, that $S \leq \overline{v}_1^B(1 - \alpha) + \underline{v}_2^B - v_2^B < 0$. Finally, if $C = v_1^B + \underline{v}_2^A$, $S = v_1^B - v_2^B + v_2^A - \overline{v}_1^A + \underline{v}_2^A - v_1^A + (1 - \alpha)\overline{v}_1^B$. Using $2.i$ and $2.iii$, we have $S \leq (1 - \alpha)\overline{v}_1^B < 0$.

Case 3: $A = \alpha v_2^A + v_2^B$

This case implies that the following inequalities hold:

$$
\begin{aligned}
\alpha v_2^A + v_2^B &\geq v_1^A + v_2^B \ (3.i) \\
&\geq v_1^B + v_2^A \ (3.ii) \\
&\geq v_1^A + \alpha v_1^B \ (3.iii)
\end{aligned}
$$

Case 3.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. The surplus can be written as $S = C - A$. Thus S, is always negative (implied by $3.i$, $3.ii$ and $3.iii$).

Case 3.2: $B = \overline{v}_1^A + v_2^B$. The surplus can be written as $S = -\alpha v_2^A - \alpha \overline{v}_1^B + C$. If $C = v_1^A + \alpha v_1^B$, then $S = v_1^A - \alpha v_2^A + +\alpha(v_1^B - \overline{v}_1^B) < 0$ (implied by $3.i$). If $C = v_1^A + \underline{v}_2^B$, then $S = v_1^A - \alpha v_2^A + \underline{v}_2^B - \alpha \overline{v}_1^B \leq \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$. For the first inequality, we used $3.i$, and for the second (17). If $C = v_1^B + \underline{v}_2^A$, then $S = \underline{v}_2^A - \alpha v_2^A + v_1^B - \alpha \overline{v}_1^B < 0$. Finally, if $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, then $S = \alpha(\underline{v}_2^A - v_2^A) + \underline{v}_2^B - \alpha \overline{v}_1^B < \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$, which follows from (18).

Case 3.3: $B = \overline{v}_1^B + v_2^A$. Then, if $C = v_1^A + \alpha v_1^B$, $S = -\alpha v_2^A - v_2^B + \overline{v}_1^B + v_2^A + v_1^A + \alpha v_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B = -\alpha v_2^A - v_2^B + v_2^A + v_1^B - v_1^B + \overline{v}_1^B + v_1^A + \alpha v_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B \leq (\alpha - 1)(v_1^B - \overline{v}_1^B) + v_1^A - \overline{v}_1^A < 0$, where the second to last inequality follows from $(3.ii)$. If $C = v_1^A + \underline{v}_2^B$,then $S = \{v_2^A - \alpha v_2^A\} + \{\underline{v}_2^B - v_2^B\} + \{\overline{v}_1^B - \alpha \overline{v}_1^B\} + \{v_1^A - \overline{v}_1^A\} < 0$; this is easy to see given that $\alpha > 1$. Now, if $C = v_1^B + \underline{v}_2^A$, $S = -\alpha v_2^A - v_2^B + v_2^A + v_1^B + \underline{v}_2^A - \overline{v}_1^A + \overline{v}_1^B - \alpha \overline{v}_1^B$, then using $(3.ii)$, $S \leq \underline{v}_2^A - \overline{v}_1^A + \overline{v}_1^B - \alpha \overline{v}_1^B < \underline{v}_2^A - \overline{v}_1^A$. Thus, $S < 0$, because of (18). Finally, if $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, then $S = \{v_2^A - \alpha v_2^A\} + \{\underline{v}_2^B - v_2^B\} + \{\overline{v}_1^B - \alpha \overline{v}_1^B\} + \{\alpha \underline{v}_2^A - \overline{v}_1^A\}$, which is less than zero because of (17), (18) and $\alpha > 1$.

Case 3.4: $B = \alpha v_2^A + v_2^B$. The surplus can be written as $S = C - \overline{v}_1^A - \alpha \overline{v}_1^B$. If $C = v_1^A + \alpha v_1^B$, then $S = v_1^A - \overline{v}_1^A + \alpha(v_1^B - \overline{v}_1^B) < 0$. If $C = v_1^A + \underline{v}_2^B$,then $S = v_1^A - \overline{v}_1^A + \underline{v}_2^B - \alpha \overline{v}_1^B < 0$, which follows from (17). If $C = v_1^B + \underline{v}_2^A$, then $S = v_1^B - \alpha \overline{v}_1^B + \underline{v}_2^A - \overline{v}_1^A < \underline{v}_2^A - \overline{v}_1^A \leq 0$, which follows using (18). If $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, $S = \alpha \underline{v}_2^A - \overline{v}_1^A + \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$ (17) and (18).

Case 4: $A = \alpha v_2^A + v_2^B$.

This case implies that the following inequalities hold:

$$\begin{aligned} v_1^B + v_2^A &\geq v_1^A + \alpha v_1^B \ (4.i) \\ &\geq v_1^A + v_2^B \ (4.ii) \\ &\geq \alpha v_2^A + v_2^B \ (4.iii) \end{aligned}$$

Using $(4.iii)$ and the fact that $\alpha > 1$, it is easy to show that $\overline{v}_1^B + v_2^A \geq \alpha v_2^A + v_2^B$ and $v_1^B + \underline{v}_2^A \geq \alpha \underline{v}_2^A + \underline{v}_2^B$. Moreover, from $(4.iii)$ is easy to check that $v_1^B > v_2^B$. Thus, $v_1^A + \alpha v_1^B > v_1^A + v_2^B$ and C can necessarily take only two values.

Case 4.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. Then, the surplus can be written as $S = C - A < 0$.

Case 4.2: $B = \overline{v}_1^A + v_2^B$. Then, v_2^B must be greater than $\alpha \overline{v}_1^B$. Thus, $\alpha v_2^A + v_2^B \geq \alpha v_2^A + \alpha \overline{v}_1^B > \alpha(v_1^B + v_2^A)$, which is a contradiction with $(4.iii)$, implying that this case is impossible.

Case 4.3: $B = \overline{v}_1^B + v_2^A$. In this case, $S = -v_1^B + \overline{v}_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B + C$. If $C = v_1^A + \alpha v_1^B$, which implies that $S = (\alpha - 1)(v_1^B - \overline{v}_1^B) + v_1^A - \overline{v}_1^A < 0$. Finally, if $C = v_1^B + \underline{v}_2^A$, then $S = \overline{v}_1^B(1-\alpha) + \underline{v}_2^A - \overline{v}_1^A < \underline{v}_2^A - \overline{v}_1^A$, which is negative because of (18). ■

Proof of Proposition 9

In this case, the surplus can be written as:

$$\begin{aligned} S(v_1^A, v_1^B, v_2^A, v_2^B) \ = \ & -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \ (\tilde{A}) \\ & + \max\{\overline{v}_1^A + \alpha \underline{v}_1^B, \overline{v}_1^A + v_2^B, \underline{v}_1^B + v_2^A, \alpha v_2^A + v_2^B\} \ (\tilde{B}) \\ & + \max\{v_1^A + \alpha v_1^B, v_1^A + \overline{v}_2^B, v_1^B + \underline{v}_2^A, \alpha \underline{v}_2^A + \overline{v}_2^B\} \ (\tilde{C}) \\ & - \overline{v}_1^A - \overline{v}_2^B \end{aligned}$$

We now establish that under (19) and (20), the surplus S is negative: In order to do that, we have to examine a number of straightforward cases:

Case 1: $\tilde{B} = \overline{v}_1^A + \alpha \underline{v}_1^B$. Then, when $\tilde{C} = v_1^A + \alpha v_1^B$ and $S = -\tilde{A} + v_1^A + \alpha v_1^B + \overline{v}_1^A + \alpha \underline{v}_1^B - \overline{v}_1^A - \overline{v}_2^B \leq \alpha \underline{v}_1^B - \overline{v}_2^B \leq 0$, by (19). If $\tilde{C} = v_1^A + \overline{v}_2^B$, then $S = -\tilde{A} + v_1^A + \overline{v}_2^B + \overline{v}_1^A + \alpha \underline{v}_1^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^A - \alpha v_1^B + v_1^A + \alpha \underline{v}_1^B \leq 0$. If $\tilde{C} = v_1^B + \underline{v}_2^A$ then $S = -\tilde{A} + v_1^B + \underline{v}_2^A + \overline{v}_1^A + \alpha \underline{v}_1^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + v_1^B + \underline{v}_2^A + \alpha \underline{v}_1^B - \overline{v}_2^B < \alpha \underline{v}_1^B - \overline{v}_2^B \leq 0$, where the last inequality stands due to (19). Finally, when $\tilde{C} = \alpha \underline{v}_2^A + \overline{v}_2^B$, then $S = -\tilde{A} + \alpha \underline{v}_2^A + \overline{v}_2^B + \overline{v}_1^A + \alpha \underline{v}_1^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + \alpha \underline{v}_1^B + \alpha \underline{v}_2^A < 0$.

Case 2: $\tilde{B} = \overline{v}_1^A + v_2^B$. Now, if $\tilde{C} = v_1^A + \alpha v_1^B$, then $S = -\tilde{A} + v_1^A + \alpha v_1^B + \overline{v}_1^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq v_2^B - \overline{v}_2^B \leq 0$, the equality holds when $v_2^B = \overline{v}_2^B$. If $\tilde{C} = v_1^A + \overline{v}_2^B$, then $S = -\tilde{A} + v_1^A + \overline{v}_2^B + \overline{v}_1^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B = v_1^A + v_2^B - \tilde{A} \leq 0$. If $\tilde{C} = v_1^B + \underline{v}_2^A$, then $S = -\tilde{A} + v_1^B + \underline{v}_2^A + \overline{v}_1^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + v_1^B + \underline{v}_2^A + v_2^B - \overline{v}_2^B \leq 0$, and finally if $\tilde{C} = \alpha \underline{v}_2^A + \overline{v}_2^B$, then we have that $S = -\tilde{A} + \alpha \underline{v}_2^A + \overline{v}_2^B + \overline{v}_1^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_2^B - \alpha v_2^A + \alpha \underline{v}_2^A + v_2^B \leq 0$.

Case 3: $\tilde{B} = \underline{v}_1^B + v_2^A$. In this case, if $\tilde{C} = v_1^A + \alpha v_1^B$, then $S = -\tilde{A} + v_1^A + \alpha v_1^B + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + v_1^A + \alpha v_1^B + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B = (\alpha - 1)v_1^B + v_1^A - \overline{v}_1^A + \underline{v}_1^B - \overline{v}_2^B < \underline{v}_1^B - \overline{v}_2^B \leq 0$ by (19). If $\tilde{C} = v_1^A + \overline{v}_2^B$, then $S = -\tilde{A} + v_1^A + \overline{v}_2^B + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B \leq -v_2^A - v_1^B + v_1^A + \underline{v}_1^B + v_2^A - \overline{v}_1^A \leq 0$. Now, in the case that $\tilde{C} = v_1^B + \underline{v}_2^A$, we have that $S = -\tilde{A} + v_1^B + \underline{v}_2^A + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + v_1^B + \underline{v}_2^A + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B = \underline{v}_2^A - \overline{v}_1^A + \underline{v}_1^B - \overline{v}_2^B \leq 0$, which follows by (19) and (20). Finally, if $\tilde{C} = \alpha \underline{v}_2^A + \overline{v}_2^B$ then $S = -\tilde{A} + \alpha \underline{v}_2^A + \overline{v}_2^B + \underline{v}_1^B + v_2^A - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + \alpha \underline{v}_2^A + \underline{v}_1^B + v_2^A - \overline{v}_1^A = \underline{v}_1^B - v_1^B + \alpha \underline{v}_2^A - \overline{v}_1^A$, which is less than zero because of (20).

Case 4: $\tilde{B} = \alpha v_2^A + v_2^B$. In this case, if $\tilde{C} = v_1^A + \alpha v_1^B$, then we have that $S = -\tilde{A} + v_1^A + \alpha v_1^B + \alpha v_2^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq -v_1^B - v_2^A + v_1^A + \alpha v_1^B + \alpha v_2^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B = (\alpha - 1)(v_1^B + v_2^A) + v_1^A - \overline{v}_1^A + v_2^B - \overline{v}_2^B < 0$. If $\tilde{C} = v_1^A + \overline{v}_2^B$, then we have that $S = -\tilde{A} + v_1^A + \overline{v}_2^B + \alpha v_2^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq -\alpha v_2^A - v_2^B + v_1^A + \alpha v_2^A + v_2^B - \overline{v}_1^A = v_1^A - \overline{v}_1^A \leq 0$, where equality holds whenever $v_1^A = \overline{v}_1^A$. Now if $\tilde{C} = v_1^B + \underline{v}_2^A$, then $S = -\tilde{A} + v_1^B + \underline{v}_2^A + \alpha v_2^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq \alpha v_2^A - v_2^A + \underline{v}_2^A - \overline{v}_1^A + v_2^B - \overline{v}_2^B < 0$, due to (20). Finally, if $\tilde{C} = \alpha \underline{v}_2^A + \overline{v}_2^B$, then $S = -\tilde{A} + \alpha \underline{v}_2^A + \overline{v}_2^B + \alpha v_2^A + v_2^B - \overline{v}_1^A - \overline{v}_2^B \leq \alpha \underline{v}_2^A - \overline{v}_1^A$, which is negative by (20)

From the analysis of the above cases, we saw that $S \leq 0$ always holds. We also saw that there is a region of types with a non-empty interior where $S < 0$. Hence, there is no feasible and ex-post efficient mechanism. If the status quo is given by (B, A), the analysis is analogous. ■

Proof of Corollary 2

For the environment under consideration, (10) reduces to

$$E\left[-(I-1)\sum_{k\in K}\max_{i\in I}\{\pi_{ik}(v_{ik})\} + \sum_{k\in K}\sum_{i\in I}\max\{\pi_{ik}(v_i^{k*}), \pi_{-i,k}(v_{-i,k})\}\right] \geq \sum_{k\in K}\sum_{i\in I} r_i \cdot \pi_i(v_i^{k*}),$$

which, in turn, can be rewritten as

$$\sum_{k\in K}\left[E\left[-(I-1)\max_{i\in I}\{\pi_{ik}(v_i^k)\} + \sum_{i\in I}\max\{\pi_{ik}(v_i^{k*}), \pi_{-i,k}(v_{-i}^k)\}\right] - \sum_{i\in I} r_i \cdot \pi_i(v_i^{k*})\right] \geq 0.$$

The expression inside the first sum is pointwise non-negative for all $k \in K$ if $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_j^{k*})$ for all $i, j \in I$. This can be immediately seen from the proof of Proposition 11, establishing the first point.

Second, the existence of optimal property rights that guarantee the existence of feasible ex-post mechanisms again mirrors the proof in Lemma 2. In fact, the condition for critical types can be rewritten as $\prod_{j\neq i} F_{jk}(\pi_{jk}^{-1}(v_i^{k*})) = r_{ik}$, and the analysis of Proposition 11 applies.

Finally, if for all $k \in K$ there exists $i \in I$ such that $r_{ik} = 1$, the expression inside the first sum reduces to

$$-(I-1)\max_{j\in I}\pi_{jk}(v_j^k) + \sum_{j\neq i}\max\{0, \pi_{-j,k}(v_{-j}^k)\} + \pi_{i,k}(\overline{v}_i^k) - \pi_{i,k}(\overline{v}_i^k)$$

which is clearly non-positive everywhere and strictly negative for the region where $\max_{i\in I} \pi_{j,k}(v_j^k) = \pi_{i,k}(v_i^k)$, establishing the last point. ■

7. Appendix B

Example: Expected Surplus is Non-Monotonic in Asymmetric Environments Consider an environment with distributions $F_i(x) = x^{p_i}$ in $[0,1]$. It is easy to see that $v_i^* = (\alpha^{2p_j+1}+1)^{-1/p_j}$ and $\frac{\partial v_i^*}{\partial \alpha} = -(\alpha^{2p_j+1}+1)^{-1/p_j-1} p_j^{-1}(2p_j+1)\alpha^{2p_j}$.

Fix $\alpha > 1$, but close to 1, and take $p_1 \longrightarrow 0$ and $p_2 \longrightarrow +\infty$. Then, with probability close to 1, $v_2 \geq v_1$ and, moreover, $F_2(v_1^*) = 1/(\alpha^{2p_2+1}+1) \sim 0$, but $F_1(v_2^*) \sim 1/2$. Therefore, only two cases appear with probability bigger than ϵ, both of them close to $\frac{1}{2}$:

$$\begin{aligned} S(v_1, v_2, \alpha) &= (1+\alpha)v_1 - v_1^* - v_2^* \text{ if } v_1 \geq v_2^* \text{ and } v_1^* \leq v_2 \\ S(v_1, v_2, \alpha) &= \alpha v_2^* - v_1^* \text{ if } v_1 \leq v_2^* \text{ and } v_1^* \leq v_2. \end{aligned}$$

Then, we have

$$\frac{\partial \bar{S}(\alpha)}{\partial \alpha} = \int_{v_2^*}^{1} v_1 f_1(v_1) dv_1 - \frac{\partial v_1^*}{\partial \alpha} + \frac{v_2^*}{2} + \frac{\alpha - 1}{2}\frac{\partial v_2^*}{\partial \alpha}.$$

Noting that the first three terms are bounded by one, but that $\frac{\partial v_2^*}{\partial \alpha}$ goes to minus infinity, we see that $\frac{\partial \bar{S}(\alpha)}{\partial \alpha} < 0$.

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